Optimization of Production Planning for a Quota-Based Integrated Commercial Fishery

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Abstract

A quota-based integrated commercial fishery owns fishing trawlers, processing plants, and fish quotas. Such a fishery must decide how to schedule trawlers for fishing and landing, how to schedule processing of products, how to schedule labour for processing, and how to plan inventory of raw materials and products. This problem is of great economic significance to New Zealand, whose economy depends to a large extent on the fishery industry. To assist the fishery manager, we develop a mixed integer linear program (MILP) for optimal scheduling of fishing trawlers, production planning (processing) and labour allocation for a quota-based integrated fishery of New Zealand. The model decides when and where each trawler should go for fishing, how much fish each trawler should land, and how much product to produce in each period. Since the fishery is a private farm, its main objective will be profit maximization (or cost minimization if its demand is on contract). The government manages the conservation of fish through the quota allocation. In this thesis the objective of the fishery model is to maximise the total profit. We demonstrate our model with examples based on data from a major New Zealand fishery.

We investigate ways to manage the uncertainties involved in trawler scheduling and production planning of the fishery. To manage end-of-planning-horizon effects in the fishery, we develop a simple safety stock approach. We also analyse the workability of a rolling horizon approach to solve the longer planning horizon models and to deal with the end-of-planning horizon effects.

We investigate the effect of initial and final position of the trawlers on the profit. We also investigated many different challenging data sets to observe the impact on the effectiveness of our IFPM.

The second objective of this thesis is to develop an efficient solution procedure for the MILP, named integrated fishery planning model (IFPM). The IFPM consists of a fishing subproblem, a processing subproblem, and complicating side constraints. We have tried techniques including LP relaxation, Lagrangean relaxation (LR), Dantzig-Wolfe decomposition (DWD) and decomposition-based pricing (DBP). We develop a new DBP method to solve the IFPM. It gives excellent computation times. We also develop a decomposition-based O'Neill pricing (DBONP) method to improve the solution obtained from DBP procedure. It improves the DBP solutions but takes longer time to solve the IFPM. Finally, we develop a simple and efficient reduced cost-based pricing (RCBP) method. It takes less time to solve the IFPM and yields excellent results.

The initial formulations for several planning horizons are solved using the AMPL modelling language and CPLEX with branch and bound. Relevant results and computational difficulties are reported.

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Chapter 1

Introduction

1.1 Integrated fishery system and its importance

The fishery industry is of significant national and regional importance to many countries. It contributes to a large proportion of the net exports of countries such as Canada, USA, UK, Norway, Iceland, Australia, Japan, Bangladesh and New Zealand.

The activities which are performed to maintain and improve fisheries resources and their utilizations are termed ‘fishery management’. Major activities for fishery management are exploratory fishing, quota allocation, gear and fleet allocation, processing, labour allocation, and marketing. An integrated commercial fishery owns fish quotas, trawlers for harvesting the fish, and processing plants to process the raw fish, in order to produce many different products. There has been growing recognition that fisheries have to be viewed as a total system from the fish in the water to the fish on the plate. This system includes fishing, trawler scheduling, processing, labour allocation, quota allocation, and marketing. These activities are made complex by uncontrollable factors such as variability in the catch rates, weather conditions, available quotas, and seasonality of fish stock availability.

This thesis is organised as follows.

In Chapter 1, we present the introduction, general background, and purpose of the thesis.

In Chapter 2, we present a literature review on fishing, trawler scheduling, processing and their co-ordination.

In Chapter 3, we present the formulation of the mixed integer integrated fishery planning model (IFPM).

In Chapter 4, we present the structure of the IFPM and discuss the computational experiments. We also examine a rolling horizon approach to decompose the IFPM.

In Chapter 5, we discuss Lagrangean relaxation, solution strategies, and algorithms, along with partial computational results. We present a Dantzig-Wolfe decomposition (DWD) approach. We develop a new decomposition-based pricing procedure (DBP) to solve our IFPM.

In Chapter 6, based on decomposition and O'Neill pricing, we develop a decomposition-based O'Neill pricing (DBONP) algorithm to improve the solutions from DBP.

In Chapter 7, we develop a simple and efficient reduced cost-based pricing (RCBP) method.

In Chapter 8, we present a summary of this thesis and outline the future work.

In Appendix 1, we present a sample AMPL model, data and run file along with sample output.

In Appendix 2, we present our paper on IFPM entitled “A mixed integer linear program for an integrated commercial fishery,” as published in the South African Journal of Operational Research (*ORiON*).

In Appendix 3, we present our paper on DBP entitled “A decomposition-based pricing method for solving a large-scale MILP model for an integrated fishery” as published in the Journal of Applied Mathematics and Decision Science (*JAMDS*).

In Appendix 4, we present our paper on DBONP and RCBP, entitled “Two pricing methods for solving an integrated commercial fishery planning model,” as accepted for publication in the South African Journal of Operational Research (*ORiON*).

In Appendix 5, we present our paper on rolling horizon, entitled “How good is the rolling horizon approach for an integrated fishery planning model?” as accepted for publication in the International Journal of Ecological Economics and Statistics (IJEES).

This introductory chapter outlines the background and importance of the fishery industry to the economy of New Zealand. It also outlines the objective and structure of the thesis. In Section 1.2, we discuss the importance of the fishery industry in the economy of New Zealand. We put the subject of fisheries in the NZ context highlighting its importance in earnings, employment, and trade, and its overall contribution to the economy of New Zealand. In Section 1.3, we present an analysis of the general background of New Zealand fishery. Section 1.4 discusses the fishery environment of an integrated commercial fishery of New Zealand. Finally the objective and significance of this research is outlined in Section 1.5.

1.2 Importance of the fishery industry in the economy of New Zealand

At approximately 2.5 million square kilometres of ocean, ranging over 30 degrees of latitude, New Zealand's main exclusive economic zone (EEZ) is the fourth largest in the world and is fourteen times larger than its land mass. This exclusive economic zone (Figure 1.1) produces just less than 1% of the world's catch. New Zealanders view this fishery industry as an important contributor to New Zealand's food supply, the health of its citizens, foreign exchange earnings, and employment.

The New Zealand Seafood Fishing Industry Council commissioned a study for economic impact assessment for New Zealand, conducted by McDermott Fairgray Group Limited, which revealed that the fishery industry is the fourth largest foreign exchange earner, worth NZ$1.7 billion in 2004. Seafood exports total $1.4 billion and domestic seafood sales are around $130 million per year. Around 27,000 people are directly or indirectly employed in the fishery industry (New Zealand Official Yearbook, 2004/05) (see Figure 1.2 (a)-(e)).

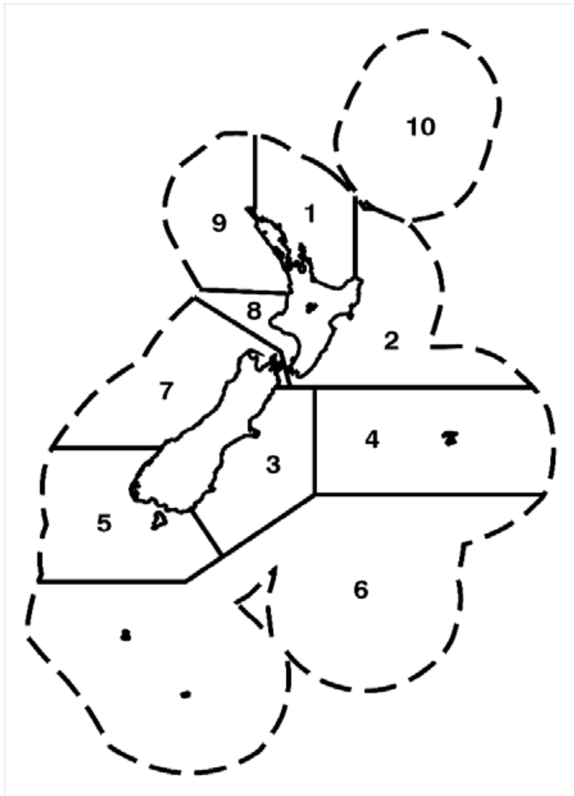

Figure 1.1: The quota management areas (QMAs) of New Zealand

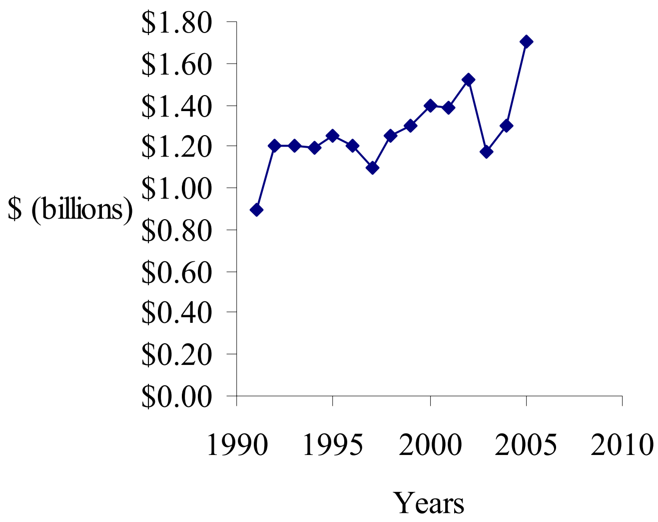

Figure 1. 2(a): Contribution to New Zealand GDP.

In figure 1.2(a), we see a dramatic decrease in seafood export in 2003. Seafood export fell about 21% in 2003 largely due to the strengthening New Zealand dollar, with only 5% growth in 2004 to offset such a loss. It increases about 20% in 2005, i.e., as dramatic as the 2003 decrease.

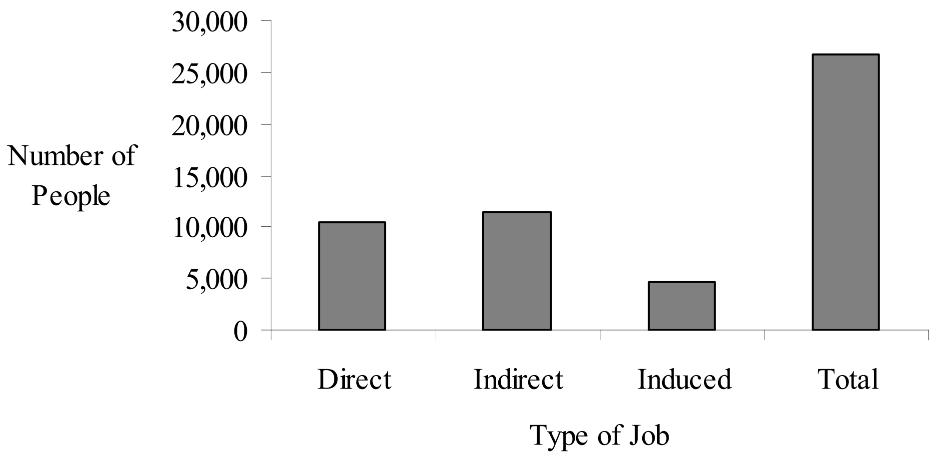

Figure 1.2(b): Number of people working in the seafood industry (Source: McDermott Fairgray Ltd.).

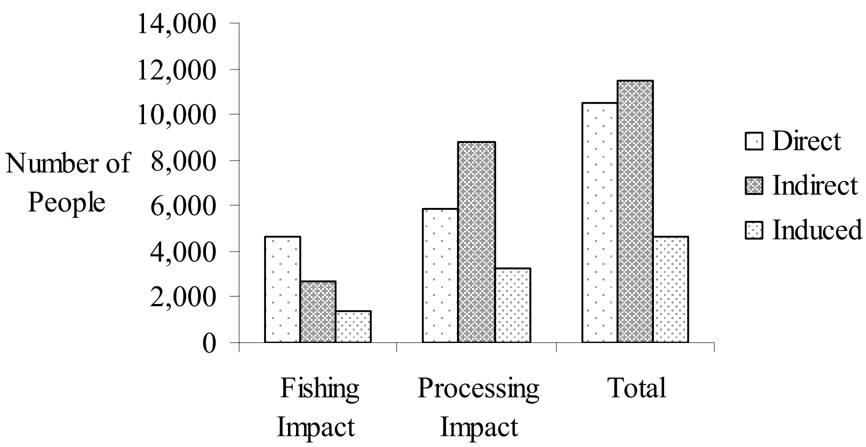

Figure 1.2(c): Number of people working in fishing and processing (Source: McDermott Fairgray Ltd.).

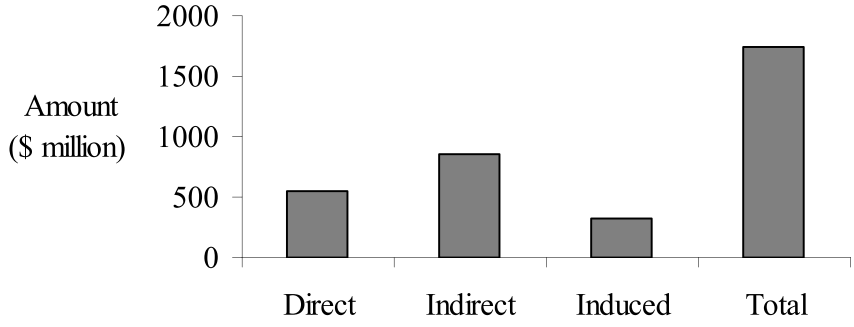

Figure 1.2(d): Contribution of Seafood industry to New Zealand GDP (Source: McDermott Fairgray Ltd.).

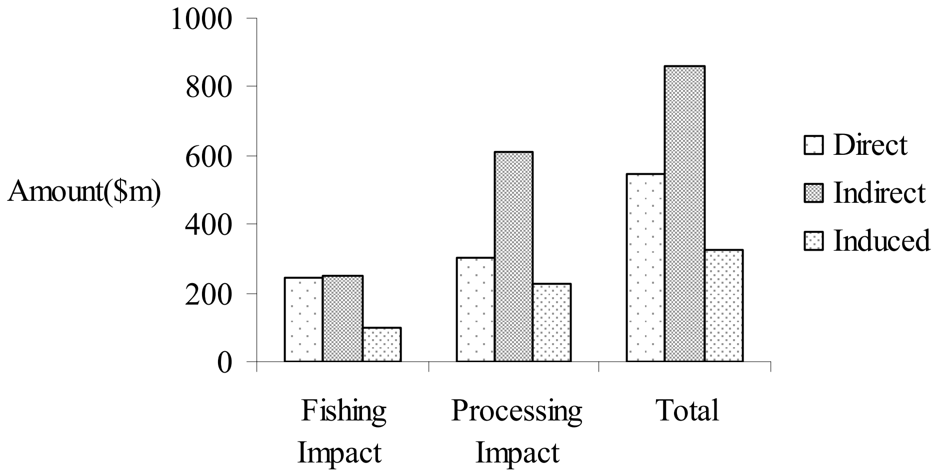

Figure 1.2(e): Fishing and processing impact on New Zealand economy (Source: McDermott Fairgray Ltd.)

Figure 1.2(b) and 1.2(d) show the number of people working directly or indirectly in the seafood industry and the contribution of the seafood industry to the NZ's GDP. Figure 1.2(c) and 1.2(e) show the number of people working directly or indirectly in the fishing and processing separately and the contribution of the fishing and processing to the NZ's GDP.

1.3 Management of New Zealand fisheries

In this section, we discuss the commercial fish species harvested from New Zealand water, the management of these resources, and quota allocation.

1.3.1 Commercial fish species

Around 130 species are harvested from New Zealand waters. According to availability, these fish species are categorized as deep-water species, middle-deep-water species, inshore species, open sea (pelagic) species and shellfish species. Mid and deep-water species account for over 50% of New Zealand's seafood export earnings. Around 70% of NZ's fish harvest is taken from deep-water and mid-water fisheries, 11% are open sea, 10% are farmed species, and 9% are from inshore fisheries. Only 43 of the 130 fish species are commercially significant. The main inshore species are snapper, red cod, sole, bluenose, and John Dory. The main mid-water species are hoki, hake, squid, and ling. Among the major deep water species are roughy, oreo dories, and silver warehouse. Hoki is the largest seafood export in New Zealand. In Table 1.1, we present the percentage of different commercial fish species exported (in weight) in 2004. Table 1.2 shows the principal seafood exports of New Zealand by value and weight from the year 2000 to 2004. From Table 1.1 and 1.2, we notice that hoki and squid are two major export species (in weight) and hoki and Rock lobster are two major foreign exchange earners.

Fish species	% export in weight
Hoki	14.1
Squid	13.9
Rock Lobsert	8
Roughy	7
Paua	4
Ling	4
Hake	4
Tuna	3
Others	46

Table 1.1: Major export species (in weight) in New Zealand (Source: McDermott Fairgray Ltd.).

Year	2000		2001		2002		2003		2004	
Species	Weight (tonnes)	Value ($m)	Weight (tonnes)	Value ($m)	Weight (tonnes)	Value ($m)	Weight (tonnes)	Value ($m)	Weight (tonnes)	Value ($m)
Hoki	74.9	310.9	83.0	345.5	75.4	308.7	64.2	229.9	50.9	174.1
R.lobster	2.8	128.9	2.2	124.3	2.2	128.4	2.2	112.9	2.1	101.5
Roughy	5.3	85.3	4.1	74.1	8.6	127.2	5.8	78.5	8.9	89.8
Ling	9.4	78.5	7.6	74.1	7.8	64.9	9.0	51.7	8.9	47.4
Paua	0.9	75.8	0.9	67.7	0.8	62.7	0.7	54.5	0.7	52.3
Squid	13.5	41.9	20.9	61.2	41.3	86.2	35.9	68.5	69.8	171.7
Tunas	14.9	41.3	7.8	46.5	8.4	42.2	10.2	32.2	15.0	35.8
Salmon	2.3	31.9	3.6	37.5	6.0	43.7	4.8	39.2	4.4	35.8
Hake	6.1	39.2	5.8	34.7	6.4	35.6	6.0	32.3	8.4	44.8
Snapper	4.6	38.9	4.1	37.4	4.3	33.7	3.8	28.8	4.1	29.0

Table 1.2: Principal seafood exports of New Zealand (Source: McDermott Fairgray Ltd.).

1.3.2 Quotas

Fish belong to no one in particular and everyone in general. Unlike other industries, few techniques are in place for participants to enjoy tenure over definable units of the fishery resource base. When everyone competes for a share of a common but limited resource, the result is a zero-sum game; one man’s gain is always another man’s loss. The risk is greatest for commercial species where the fish are valuable and the cost of extraction is reasonably low. Other species may be protected simply because they are economically unattractive to catch. Fisheries worldwide continue to suffer from the negative consequences of this open access. To combat the common property phenomenon, quota allocation system is proved to be very essential and important.

To control the continuous decrease in fish supplies, the Icelandic government introduced quota regulation in 1984, but implemented it for only nine main commercial species. This system was made uniform across species in 1990.

Following the declaration of its 200-mile exclusive economic zone (EEZ) in 1978, New Zealand began to develop the fishery resources of the new zone and to restructure its fisheries management system. The 1980's brought dynamic changes in the approaches to management of New Zealand's fisheries. The government embraced an explicit objective of maximizing sustainable economic efficiency by adopting an individual transferable quota (ITQ) system as the preferred system of management for most fisheries in 1986. Currently, this program applies to 43 species in 10 management areas of New Zealand. Some motives were to avoid over-fishing, to provide an incentive to control overcapitalization in commercial fisheries, to promote conservation of stocks, to improve market conditions, to promote safety in the fishing fleet, and to improve the overall economic efficiency of the fishing industry. This approach to fisheries management has been widely praised (Straker et al, 2000).

Iceland and New Zealand lead the world in establishing the individual transferable quota systems. Other countries that use individual transferable quota systems include Australia, Canada, Italy, the Netherlands, Bangladesh, Norway, Japan and South Africa.

A quota is a permit to fish a specified amount of a particular stock in a given period, usually a year. The quotas can be issued for free, against a fee, or at a public auction to companies or individual vessels. In case the quotas are issued and not auctioned, the allocation is based upon a specified reference called the *quota base*.

In any case it is assumed that the quota-base is among the basic characteristics of the firm. Under this quota system, the firm has no need to harvest its fish before they are caught by someone else assuming all fish are uniformly distributed throughout the planning horizon. Thus a fishery can plan the best timing of its fishing activities. If the stock assessment shows that any species' stock is declining, the species quota is reduced accordingly. If the government sees that it does not have enough information about a certain fish species, again the quotas are reduced proportionally. The annual catch quotas are proportional to these quota bases, which can be a function of the size or type of the vessel, its crew size, or the vessel's previous catch record.

Quota systems are divided into total quota systems and individual quota systems. The individual quota systems can be individual non-transferable quota systems and individual transferable quota systems (ITQ).

The Ministry of Fishery introduced a quota management system (QMS) in 1986. Since then, New Zealand's main fisheries have been managed under this system which divides the 200 nautical miles of EEZ of New Zealand into ten quota management areas (QMAs). Each fish stock is defined by an area that may be the same as a QMA or a grouping of QMAs depending on the geographical distribution of that fish stock. For example, the fish species Hoki has one stock HOK1 which incorporates the QMAs 2, 5, and 7 while Ling has a stock called LIN3, which can be found only in QMA3.

1.3.3. Quota allocation

For each fish species, there are 100 million quota shares. An individual quota share entitles that fishery or individual vessel to 1/100,000,000 of the total allowable commercial catch (TACC). The quota weight equivalent (QWE) is the total amount that a share owner is allowed to catch. For example, if TACC of red cod is 16,073,000 kilograms, then the QWE = $\frac{16,073,000}{100,000,000}$ kilograms.

The TACC for a fishery is QWE × number of shares owned by that fishery.

Species	Code	Total TACC (000 Kg)	ITQ (000 Kg)	% of TACC
Hoki	HOK	200,010	15,300	7.65
Orange Roughy	ORH	15,921	5,100	32.03
Red Cod	RCO	16,074	2,800	17.42
Ling	LIN	21,977	5,500	25.02
Squid	SQU	127,332	40,000	31.41
Barracuda	BAR	32,672	10,300	31.52
Hake	HAK	12,366	3,100	25.07

Table 1.3: Sample data for a fishery's quota allocation.

In Table 1.3, we present a sample quota allocation of some fish species for a fishery.

In Table 1.4, we present a sample data for total allowable commercial catch (TACC), and the amount of fish caught from different quota management areas (QMAs) for some fish species in a fishing year. We notice that for both the fish species shown in the Table 1.4, the amount of catch is around 95% of TAACs.

Species	Stocks	TACC (Kg.)	Amount caught (Kg)	% Caught
Hoki	Hoki1	200,000,000	195,713,000	98
	Hoki 10	0	0	0
Orange Roughy	ORH1	1,400,000	1,294,000	92
	ORH10	10,000	0	0
	ORH2	1,285,000	1,267,000	98
	ORH3	12,921,000	11,724,000	91
	0RH7	111,000	95,000	86

Table 1.4: Sample data for TACC and amount caught for some fish species according to quota management areas (QMAs).

New Zealand is currently a world leader in fisheries management and supports an industry-based on sustainable harvest and environmental principles. Industry pays for all fisheries management, enforcement and research and development. It invests over 2% of gross returns into research and development; much of this comprises environmental studies. However, the advent of quota allocations has yet to be met with appropriate planning models which incorporate fishing, and processing under the environment of quota allocation. That is, the integration of fishing and processing has received less attention by researchers and authorities so far. The complexity of the fishery system and the trawler scheduling problem, and the high degree of uncertainty in catch size, weather and other factors, have contributed to the insufficiency of research in this area.

In recent years, improved seafood storage and handling techniques have improved export returns, as have developments in value added products. Improving storage techniques for live and fresh seafood is particularly important because of the large distances to New Zealand’s markets, mainly Japan, USA, EU, Hong Kong, Australia, China, Korea, Singapore, Taiwan, and Thailand. To take advantage of improved storage techniques, companies need OR models which co-ordinate fishing, trawler scheduling, processing and marketing.

The industry is now anticipating further growth–not through increased catch, but through growing the value of its products. In 1999, the Ministry of Fisheries adopted an ambitious vision to improve export fishery products in the world markets. The vision is "To be the preferred supplier of high quality fish product to discerning world seafood markets." It anticipates total export returns of $2 billion by 2010, and in order to achieve, it is important to develop scientific methodologies for fishing, trawler scheduling, processing and labour allocation.

1.4 An integrated fishery environment

In this section, we describe the various components of an integrated commercial fishery.

1.4.1 Fish

Fish is a highly perishable raw material. To avoid spoiling, fresh fish must be processed quickly. It needs proper care from the time it is caught until it is served or delivered for processing. The handling of fresh fish during this interval determines the extent to which deterioration takes place from enzymatic, oxidative and bacterial action.

1.4.1.1 Fish stock

A fish stock is a population of a particular fish species which inhabits a particular area. It is a group of fish that can be treated as a homogeneous and independent unit. A few members of that group may mingle with other groups but most stay with their own group.

1.4.1.2 Quality of raw material (fish)

The quality of raw material influences the products that can be produced from landed fish, and consequently quality influences potential revenue. The quality of fish varies with season, spawning conditions, moulting conditions, age, fishing grounds, etc., and those characteristics influence especially the fat content of the flesh, the degree of saturation of the fatty acids percent in the fatty fish and the flavour of the flesh. Based on the quality of the fish landed, the firm must decide, in the light of market requirements, the set of products to produce over the planning horizon. For planning purposes, landed fish is classified into quality types. The fishery measures the quality of raw material by visual inspection and smell, looking at the eyes and gill, temperature of the fish and by the time the trawler has been out etc. If a trawler has been out longer (e.g. say 10 days) the quality of its fish will be worse than that of from a trawler has been out for 5 days. In our model, we classified the type of raw material in three quality groups: type 1, type 2, and type 3 with type 1 being of the highest quality and type 3 being worse. In example IFPMS, we use just two quality types; acceptable and unacceptable. The unacceptable raw material is used to make into fish meal.

1.4.2 Fishing Trawlers

Stern trawlers and mid-water trawlers are used for fishing. The fleet may be homogeneous or heterogeneous. A heterogeneous fleet will have varying capacities, speeds and operating costs. According to the fuel consumption, streaming speed, storage capacity, operating cost, etc., the fishery classifies the fishing trawlers into trawler classes. The integrated commercial fishery studied here owns two classes of trawlers. The trawler's streaming speed is the speed by which the trawlers go to the fishing ground and come back to the port.

The trawler's catch rate is defined as $\dfrac{\text{Amount of caught fish of a quota stock}}{\text{Days of fishing effort}}$. Catch rate in a fishing area is variable. This variability is a function of the stock, the season of the year, fish population, the skill of the captain, the type of vessels, and weather conditions. Thus, catch rate is a parameter requiring serious attention by fishing firm. The amount of catch rate is an estimate of the catch rate expected if fishing activities were undertaken at appropriate times during the year. In our model, the catch rates used are the best guess estimates provided by knowledgeable personnel in the industry. Company data currently available, with some assumption was used to develop catch rate distribution as a function of time, trawler class and fish stock.

While fishing for a particular species, a trawler often harvests other species as by-catch. It is assumed and expected that for most stocks the principal species can be caught with minimal by-catch. If the quota for the species in the by-catch has already been exhausted, then the fishing in that region becomes illegal. Then to fish from that area, the fishery can lease or swap quota of that fish species. The fishery also can pay a "deemed value," basically a fine to catch fish from that area.

A trawler's trip is the movement of a trawler for the purpose of fishing, from any landing port to a set of distinct fish stocks, and again from those stocks to the landing port. The beginning and ending ports need not be identical. In this thesis, we consider the beginning and landing ports identical. Though occasionally the initial and landing port in NZ are not identical (for example, Hoki is sometimes unloaded in Piction rather than Timaru), this is not a sever limitation of the model, as fish is trucked back to the processing plant, unless it's between islands.

The cost of a trawler's trip is calculated by summing the cost of fuel consumption during fishing and streaming, daily operating cost due to crew salaries, and gear maintenance cost.

The cost per kilogram of landed fish can be calculated as

$$\frac{\text{Cost of a trip}}{\text{Total kilograms of fish landed}}.$$

To guard against the landing of poor quality fish, the fisheries restrict the length of a trawler's trip. The length of trawler's trip depends on the storage capacity of the trawler and the catch rate of different fish species. As we will see in Chapter 4, if the storage capacity of the trawler is reduced, then the trip length decreases. If the storage capacity of the trawler is increased, the length of the trip increases. Again we will see in Chapter 5 that, if the catch rate increases, then the length of trawler's trip decreases, because it takes shorter time to reach the capacity of the trawler. As a result the profit of the fishery increases. Again, if the catch rate decreases, then the length of trawler's trip increases, because it takes longer time to reach the capacity of the trawler. As a result the profit of the fishery decreases. Also the quality of the raw fish decreases with the longer trawler's trip.

1.4.3 Fish Processing

The word “processing” is used ambiguously in the fishing industry. It normally refers to any operation performed on fish. Fish processing produces alternative products from the same raw material (fish species). Fish processing by trawlers at sea usually involves chilled storage of the fish in crushed ice until the vessel returns to the port. When the trawler arrives at the freezing plant, the fish are inspected and graded by size and quality. The fish are unloaded, transported to the processing plant, and then processed according to the type and quality of the fish. At the plant, processing operations include cleaning, cutting, filleting, wrapping, forming, coating, grinding, drying, packing, and freezing. Some fish species need skinning operations for processing. The processing steps of different products are shown schematically in Figure 1.3.

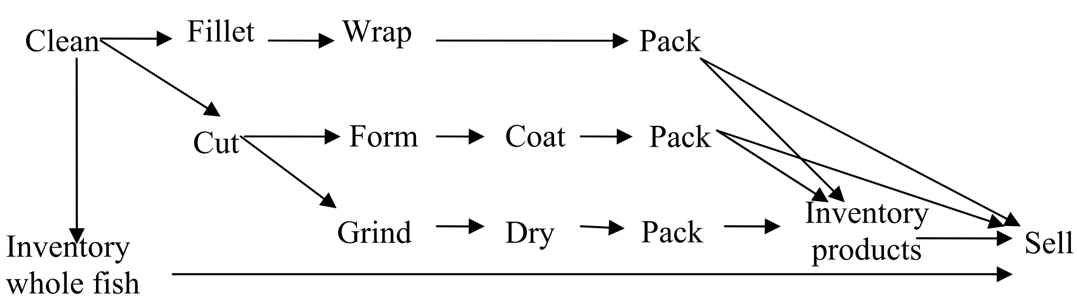

Figure 1.3: Processing steps of different products.

1.4.3.1 Quality of product

Depending on the quality, Gunn and Newbold (1987) identified five broad categories for classifying fish products. These categories are (i) fresh round fish, (ii) fresh fillets, (iii) products made from fish with < 25% broken flesh, (iv) 25% broken flesh < products made from fish with < 50% broken flesh and (v) products made from fish with > 50% broken flesh. Fresh round fish and fresh fillets fetch the highest prices and can be made only from landed fish of the highest quality. Fish of the highest quality can be converted to any product category in accordance with a yield factor. As the quality level decreases, the range of possible products is diminished. Following the quality type of raw fish into type 1, type 2, and type 3, in our model, we classify product quality into type 1, type 2, and type 3.

1.4.3.2 Type of products

The firms produce a large number of distinct products. Major products include filleted, gutted, headed and gutted, dressed, fish sticks, fish blocks, etc. Heads, offal, etc., from the fish are converted to fish meal in some plants in New Zealand. The fishery produces about 200 products in total. Products considered in this study are discussed below.

Filleted (FIL/SKF): Fillets are the flesh cut away from either side of the body of the fish from immediately behind the head or the pectoral fin to the tail. The skin may be "fillets skin on (FIL)" or "fillets skin off (SKF)". Fillets go through cleaning, filleting, wrapping, and packing.

Gutted (GUT): Gutting fish involves the removal of only the internal organs of the body cavity, whether or not the gills have been removed. A gutted fish has a longer storage life than a whole fish, because entrails cause rapid spoilage. This product goes through cleaning, cutting, forming, coating, packing, and freezing.

Headed and gutted (HGU): In addition to gutting, the processor removes the head and that portion of the body immediately forward of the pectoral fin, whether or not the tail has been removed at a point behind the posterior base of the anal fin. This product goes through the steps of cleaning, filleting, grinding, drying, and packing.

Dressed (DRE): Dressing fish involves the removal of the head and gutting of the fish. The tails, fins and the collarbone immediately behind the head are not cut off. The eggs from the female fish are generally removed for further processing, and the milt of the male fish may also be removed at this stage. Dressing goes through cutting, cleaning, gutting, and freezing.

1.4.3.3 Product Mix

The product mix refers to fish types, the list of products, and flexibility matrix. The flexibility matrix shows which product to be produced from which types of fish. That is, the product-mix problem of production planning is to determine the best quantity of each product to manufacture, over a complete range of products competing for a number of limited resources. Figure 1.4 shows a flexibility matrix for the product mix problem of quality type 1 raw material. If fish *i* is used to produce product *j* then put a "1" in raw *i* and column *j*, otherwise zero. For example fillet cannot be produced from squid. So we put a zero at the cross of squid and fillet.

	Product (*j*)		
Fish (*i*)	Fillet	HGU	GUT
Hoki	1	1	1
Squid	0	1	1
Roughy	1	1	1

Figure 1.4: A sample product-mix flexibility matrix of quality type 1 fish.

Fishery production management is constantly confronted by the product-mix problem, where it has to decide how much to produce of each of a range of items that can be manufactured by the processing firm. Even when a product mix is determined to best satisfy some criterion of performance, it is not then fixed once for all. Alterations to the mix may be required because of changes in demand, supply, costs, selling prices, or the availability of plant and labour. Some changes may be short-term and temporary; others may be long-term and permanent.

1.4.3.4 Processing factory

The processing factory is the place where the raw materials are processed to produce different products. The location of a processing plant impacts the cost of transportation from stocks to distribution centres. Plants may have a home-fleet of trawlers or not. A plant may be mechanized or only labour intensive. The fishery we considered owns trawlers, a processing firm at Timaru, and fish quotas. The processing firm can process any fish species or product, and any trawler class can land its catch there. The firm's processing and inventory storage for raw materials and products have certain limits because of availability of raw material, labour and market demand.

Scheduling fishing and processing separately will lead to suboptimization of the total system, because production planning in processing depends on a steady supply of fresh raw material from the fishing trawlers to the processing firms. Also, to promote fresh fish, fish products, and good quality frozen fish and products to the consumer, the raw material has to be delivered to the processors in a good quality condition.

1.5 The objective and significance of this research

Thus, we see that planning for an integrated fishery is of great economic significance to New Zealand, and we see the importance of integrating trawler scheduling, processing, and labour allocation, for a quota-based integrated commercial fishery.

The objectives of this thesis are two fold:

(i) to develop a mathematical model to assist the fishery to make decisions,

(ii) to find efficient solution procedures of the fishery model.

We also provide the evidence for the effectiveness of the model and the approaches developed in this thesis.

(i) In spite of the vital role of scheduling of trawlers in an integrated fishery, the co-ordination of the trawler scheduling and production planning has not attracted adequate attention of researchers in the past. This lack of attention may be due to the complexity of a large fishery and its trawler scheduling problem, and the high degree of uncertainty in catch size, weather and other factors. On the other hand, other New Zealand sectors such as electricity, timber planting and processing are enjoying the benefit of Operations Research models (Read *et al*. 1987, Read, 1992, and George & Read, 1990). But these sectors are quite difference. The relevant parallel is that NZ was the first country to introduce a smart market of electricity and also one of the first to introduce quota allocations for fisheries. Both of these are market-oriented ways of managing a national resource, taking into account necessary constraints. In both the cases, the prior method of management was to by heavy government regulation.

This dissertation is concerned with integration of fishing trawler scheduling, production scheduling (processing), and labour allocation for a quota-based integrated commercial fishery. This thesis proposes a new planning model, IFPM, optimizing two stages of the fishery supply chain independently. The first stage concerns the choice of where, when and what to fish. The second stage concerns the management of inventory and production at fish processing plants. The implementation of this model will allow a manager of the fishery to observe the expected consequences of his decisions before those decisions are implemented.

The objective of the model is to maximise the total profit of the fishery, to decide when and where each trawler should go for fishing, how much raw material of each species should be landed by each trawler from each stock and when, what amount of each product should be produced in each period, and how many regular and overtime labour hours are required for each period per trawler and the processing plants.

(ii) Since the resulting mathematical formulation of the mixed integer IFPM is difficult to solve, due to the problem size and the integrality constraints, another objective of this thesis is to find a way to solve the model efficiently. For this, we investigate the linear programming (LP) relaxation, Lagrangean relaxation (LR), Dantzig-Wolfe decomposition (DWD) and decomposition-based pricing (DBP). We choose these methods to gain into the effectiveness of these methods as a mechanism to solve the IFPM. We found that the LP relaxation, Lagrangean relaxation and subgradient optimization was weak to be effective, and DWD was unable to solve longer planning horizon models.

We develop a new DBP method to solve our mixed integer IFPM. It takes less time to solve the IFPM and yields better solution than the SO and DWD. We then develop a decomposition-based O'Neill pricing (DBONP) method to improve the DBP solution. It improves the solution obtained from the DBP but takes longer time to solve the IFPM. Finally, we develop a simple and efficient reduced cost-based pricing (RCBP) method. It takes less time to solve the IFPM and yields excellent results.

Chapter 2

A Review of OR Models for Production Planning in Integrated Fisheries

2.1 Introduction

This chapter reviews the Operations Research (OR) literature on fishing trawler scheduling, processing, labour allocation and quota allocation for integrated fisheries.

Due to the complexities in fishery problems, the fishery industry received little attention by the Operations Researchers before the 1980's. Since then, researchers have used range of models for the fisheries industries. Most of the papers described biological models, and only a few discussed production planning.

The remainder of this chapter is organized as follows. In Section 2.2, we discuss the existing OR models for fish stocks and quota allocation. Section 2.3 discusses fishing fleets and scheduling. Section 2.4 describes models related to fish processing, and co-ordination of fishing and processing. In Section 2.5, we focus on the future direction of research.

2.2 Fish Stocks and quota allocation

In this section, we review the relevant literature on fish stocks and quota allocation.

2.2.1 Fish Stock

A fish stock is the population of a particular fish species which inhabits a particular area. Most of the related literature includes models of the fluctuating fishing stock, i.e., developing population-level models. For example, Helgason (1981) presented a bio-economic dynamic deterministic optimization model of the Icelandic cod stock. The model described fishing effort, fishing mortality rate measurement and selection of age pattern of fish. The fishing mortality rates were controlled by two coefficients one measuring efforts and the other the size-selectivity of the fishing. The objective of that model was to maximize the present value of the future net profits of the fishing over an infinite planning horizon. The author solved his model in two steps. One is recursive computation of the shadow prices and the other is the optimization of the Hamiltonian expressions for each year. Their work applied a mathematical model in some detail to a specific situation in fisheries management of the Icelandic cod fishing in the next few years. The author outlined to extend their model for including other fish species for both biological and economic reason. In our model, we will harvest about eight fish species at each planning horizon.

Clark (1985) developed a dynamic model which discussed the deterministic problems in population dynamics, stock recruitments, stock exploitation, and multiple species interactions. The model used an objective function to obtain optimal decision. The state variable in the optimal harvesting context was the population dynamics system and the basic decision variable was the fishing mortality (the rate at which fish dies during the fishing effort) and harvest allowed. But for the complicated problems where the decision variables are broadened to consider age-specific harvests each year and the population dynamics, the fishery management system includes stochastic elements. Clark and Kirkwood (1986) analyzed optimal harvesting policies under uncertainty from both natural fluctuation in the stocks and uncertain elements of the stock abundance.

Quinn II and Deriso (1999) studied quantitative models of fish population dynamics and methods of fisheries stock assessment. The authors analyzed different statistical and biological models on fish population growth, mortality and fishing processes. Bjorndal et al (2004) reviewed biological and bio-economical models for fisheries fish population dynamics, fish stock assessment etc. He put emphasis on the application of the Operations Research models for the management of renewable natural resources. Azadivar et al (2002) developed a simulation-based optimization technique for estimating the population dynamics of the sea scallop population. The objective of their LP model was to maximize total fishing yield over the planning horizon over a year, subject to fishing capacity constraints, gear restrictions of trawler, and fishing area specific restrictions. To accomplish this, the authors developed fishing capacity restrictions, gear restrictions, and area specific restrictions. He combined the simulation model with an optimization model which provided a better methodology for estimating the population dynamic of fish species. The author highlighted to focus on improving the biological information on spatial and temporal scales, examining the stock-recruitment relationship, and expanding the optimization modelling effort.

2.2.2 Quotas

Helgason and Olafsson (1988) presented a deterministic decision support system for long and short term management of Icelandic fisheries. They considered the trawler type and size, temporary bans on fishing a particular fish stock, mesh size regulations of nets, and the catch quota allocation. They calculated the earnings and costs in the fisheries. The model computed the expected catch, economic outcome and other statistics by year over a 10 year planning horizon. They kept the fishing fleet, the recruitment as constant. This method proved ineffective as it did not provide incentives to vessel owners to change the fleet configurations for fishing, which at the time was estimated to be 40 percent larger than necessary (Gylfason, 2002).

Millar (1995) developed a tactical linear programming model for allocating annual surveillance effort to monitor and control fishery activity and to enforce fishery regulations in accordance with resource management plans and objectives for an offshore fishery. The objective of their model was to maximize the total value of surveillance effort allocated to the fishing grounds, subject to the constraints of total surveillance effort allocated in each period and each fishery, resource capacity, inspections and annual budget. The author employed the LP as a tool so as to allow the use of the sensitivity analysis to gain insight into the problem structure.

Arnarson (2000) developed a class of fisheries models referred to as endogenous dynamic models. The models consist of two fundamental components: (i) a biomass growth function and (ii) an economic performance function. The objective of that paper was to maximise profit at each point of time subject to relevant constraints of the fisheries along with fish quotas. In their model, each firm solves its own maximization problem subject to its own constraints and opportunities. The aggregate profit was determined by summing over different firms' profits. His models identified the limitations of bio-economical models in fisheries such as the models do not include a link between the available fisheries management measures and the course of the fishery. The advantages of his models are: (i) these models can be used to as laboratories for control experiments on the impact of specific fisheries management measures and (ii) they can be used to investigate the extension of loopholes in fisheries management.

Meester et al (2001) developed an integrated simulation model that builds upon traditional fishery management methodology by utilizing knowledge from the fields of computer simulation modelling and Operations Research. The model addressed the spatial dynamics of fish resources and human uses. To analyze fish stock distributions, fishing mortality, and fish movement, the authors integrated a population dynamics simulation model, graph optimization theory, a recursive clustering algorithm, an integer program and a linear program. The authors showed how these models can provide a robust quantitative framework for addressing a range of spatial management decisions in fisheries. The graph optimization theory was based on the idea of modelling real world spatial problems as graphs and using techniques that manipulate those graphs to determine optimal solutions of the problems. They reported a case study of Florida Keys National Marine Sanctuary (FKNMS). They first used a recursive clustering algorithm to form all possible feasible marine reserves within FKNMS. Then they used an integer program to isolate reserves that were neither adjacent nor overlapping.

2.3 Fishing fleets and fishing trawler scheduling

In this section, we discuss the application of Operations Research in fishing fleets and trawler scheduling for fishing and landing.

2.3.1 Fishing Fleets

A few OR models have been used in describing fishing fleets. For example, Jensson (1981) presented a simulation model which analyzed fleet operation and congestion problems. The author discussed the effect of fleet operations on the total catch, on the utilization of different factories and on the different size categories of boats. Jensson (1982) presented a fleet mix model describing the fishing fleet, trawler mix and trawler allocation. These models were for the operations of trawlers for fishing rather than the scheduling of trawlers for fishing.

Digernes (1982) adopted an analytical approach for single trawler operations. The author expressed revenue as a function of the operations of the trawler depending on fishing time, amount of fish gear used per fishing day, catch per unit gear used and fish price. The various cost components were associated with operational factors. For example, fuel costs were expressed as a function of engine power and operating time. This model was also for the operations of trawlers for fishing not for the scheduling of trawlers for fishing. His model computes the profitability of trawler operations for a fishing trip.

Charles and Yang (1990) developed a bio-economic dynamic model for exploring fishery development and management options subject to the realistic industrial and behavioural constraints. Their paper highlighted the complex, dynamic interactions in a coastal fishery's domestic and foreign fleets. Their simulation model addressed the role of optimal allocation of domestic versus foreign fishery rights, and the optimal dynamics of investment in domestic and foreign fleets. The model played an important role to determine what level of domestic and foreign fleet's capacity should be deployed and maintained because this is important for optimizing the use of the fishery resources.

Though the above papers played an important role for the operations of a trawler for fishing, but they did not schedule trawlers for fishing. In this thesis, we will model the optimal scheduling of fishing trawlers.

2.3.2 Fishing trawler scheduling

Fishing trawler scheduling is a well known problem in the fishery industry. Production planning in fisheries heavily depends on a steady supply of raw materials from the trawlers to the processing plants. So the production manager needs a trawler schedule which will specify when each trawler will land its catch and what amount of fish will be landed. These scheduling problems are more complicated than the traditional merchant trawler scheduling problems, because the scheduling of fishing trawlers is influenced by catching capacity and processing capacity. The distance of the fishing ground from the processing plant may vary widely, and may impact catch preservation and fuel capacity considerations.

Only a few OR papers have discussed trawler scheduling for fishing. Millar and Gunn (1990) designed a simulation model for assessing fishing fleet performance. The objective of their model was to assess the impact of catch rate uncertainty and single step real time decision making on fleet performance. Their network-based simulation model was a representation of the dispatching process which took place.

Millar (1996) presented a pure 0-1 integer programming model for planning scouting activities in fisheries. Assuming a single dispatching port and one or more trawlers, the author introduced an upper bound on the number of stocks in an optimal tour of a trawler. It enabled the model to reduce the size of the problem and consequently reduced the computational effort to solve the problem. The model maximised the total value of the scouting trips over one or more trawlers subject to different constraints on fish stock, trawler capacity, number of trips, trawler's flow in and flow out, and total budget and time on each trip. The author also discussed the importance of decision support for the fishery industry. However, if a trawler can be dispatched from several ports, the model needs to be modified to accommodate the multiple ports.

The above two papers modelled trawler scheduling for fishing. To our knowledge there is no other paper which models trawler scheduling for fishing. In our model, we will model trawler scheduling for fishing and landings and also processing, labour and quota allocations.

2.4. Processing and co-ordination of fishing and processing

Production planning and scheduling in integrated fisheries are concerned with the acquisition, utilization, and allocation of production resources to best satisfy customer requirements at minimum production cost or maximum profit. In this section, we review papers which have modelled the co-ordination of fishing and processing.

2.4.1 Fish processing

Fish processing models have been discussed in several papers. Mikalsen and Vassdal (1981) developed a multi-period LP model for one month production planning in Norway. They assessed unprocessed fish that could be frozen and stored for several months, and then thawed and processed. The authors discussed a monthly production planning model for smoothing the seasonal fluctuations of fish supply. The authors discussed the difficulties of laying off workers during the low season, the advantages of freezing technology, and product mix reflecting consumers' demands. But freezing was not acceptable in all countries. For example, in Icelandic freezing plants, freezing would lower product quality (Jensson, 1988) and so is in NZ's freezing plants. Mikalsen and Vassdal's model was market-driven and incorporated the acquisition of raw material purchased, rather than acquired with their owned fishing fleet.

Jensson (1988) developed a product mix LP to maximize the firm's profit over a five day horizon of an Icelandic fish processing firm. The model determined product mix and labour allocations. At the time of their research, the price for fresh fish in Iceland, unlike many other western countries, was fixed during a season by negotiation. The paper analyzed the production planning, market fluctuations and randomness of raw materials for fish processing firms. He also discussed the experience of real life testing. The model had approximately 160 variables, 80 restrictions and about 60 simple upper bounds.

The multi-period LP model of Mikalsen and Vassdaland (1981), the product mix model of Jensson (1988), and other similar papers did not take into account catch quota allocations. The quota is controlled by the total yearly catch, and each fishing trawler's share is based on its catch history over the previous couple of years. The randomness of the catch, increased competition of fishing, and the quota allocation forces the processing plants to schedule resources effectively to control production. Thus it is important to develop a model to include the quota allocation in the product mix model. However, the above LP models require accurate information about incoming raw materials over the planning horizon, but obtaining precise information for such variables is difficult.

2.4.2 Co-ordination of fishing and processing

There is still little reported work in co-ordinating fishing and processing.

Jonatansson and Randhawa (1986) developed a network-based simulation model of a fish processing firm. In their model, fish were modelled as entities in the system, and the machines, operators, and materials handling equipment as resources. To allow the user to input parameters of the system or to change default parameters, the authors designed a user interface to the model. Their model provided a tool for integrating complex processes with multiple workstations involving various uncertain factors. The output from their simulation model consists of statistics for various measures. The output was the products from two types of fish species; cod and catfish.

Jensson (1990) proposed a mixed integer linear program to solve the co-ordinated scheduling problem of trawler landings and plant operations. He considered the co-ordination of two weeks of operations of an Icelandic cod fish processing firm's fleet and plants. The author ignored uncertainties about future catches by assuming the average expected catch. The paper does not consider the trawler scheduling problem. The author, however, suggested combining the product mix and manpower allocation model to make it more realistic. However, the production manager of a fish processing firm needs an initial schedule for trawler trips, along with the amount of raw material that each trawler lands.

Gunn, et al. (1991) studied tactical planning for a Canadian company with integrated fishing and processing. The authors formulated an LP to determine the product mix so as to maximize profit. The model included a fleet of trawlers, a number of processing plants and market requirements. Their model consists of a fishing plan specifying which stocks to fish but not when and where, a marketing plan specifying which types of products to sell and inventory and production plans for the plant. Their paper discussed issues for further research. However, their model ignored the trawler scheduling problem, the stochastic nature of operations, and the quality-time relationship which affects the value of fish products.

Millar and Gunn (1992) developed a two-stage procedure for planning marketing and fishing activities for fish processing firms. Their approach comprised two decision frameworks for harvesting and marketing in integrated fish processing firms. The first stage employed an LP to develop an initial annual tactical plan (Gunn et al 1991). That stage addressed the aggregate use of the firm's resources over a one year planning horizon. The second stage of the hierarchical planning methodology involved short-term decisions concerning the dispatching and routing requirements at minimum cost. The author used the decision obtained from stage one to set the parameters and boundaries for the second stage for trawler scheduling.

Randhawa (1994) integrated an LP and a simulation for co-ordinating fishing and processing. He determined a trawler's fishing schedule and generated the quantity of catch during the fishing trip using a simulation model. He used an LP to determine the allocation of raw material and labour, mix of products, and inventory of raw material. The author put emphasis on co-ordination between fishing operation on the sea and the processing at on-shore fisheries.

None of the papers discussed in this chapter took into account the interaction among trawler scheduling, processing, quota allocation, and labour allocation. Since production depends on the steady supply of raw materials from the fishing fleet, trawler scheduling plays an important role, so it is important to develop a unified model to address trawler scheduling and processing.

2.5 Conclusion

In this chapter, we reviewed some relevant OR models in fish stocks and quotas, fishing trawler scheduling, processing, and labour allocation. We stated the importance of the co-ordination of fishing and processing. Although there are many articles on optimization of fisheries, only a few discussed catch quota allocation. A quota-based integrated commercial fishery needs an integrated model which will address fishing, trawler scheduling, processing, quotas and labour allocation comprehensively. To co-ordinate these stages in the supply chain, trawler scheduling can play an important role. None of the above mentioned papers addressed these planning issues comprehensively.

The emergence of global, competitive markets has increased the need for efficient production processes, to reduce investment and operational costs and to increase productivity. Better management practices can play an increasing vital role. The increasingly complex environmental issues present additional challenges for Operational Research models. In this context the significant role of OR models and methods for the understanding and management of renewable natural resources seems unquestionable.

To address all of the above issues, i.e. fishing, trawler scheduling, processing, labour allocation for processing, and quota allocation, we will develop a mixed integer fishery planning model in the following chapter.

Chapter 3

A Mixed Integer Linear Program for an Integrated Fishery

3.1 Integrated fishery planning (IFP)

In this chapter, we develop a mixed integer linear program (MILP) to help to co-ordinate trawler scheduling, processing, and labour allocation of a quota-based integrated commercial fishery. The aim of this model is to help the manager of the fishery to decide when and where a trawler should go for fishing, how much raw material could be landed, how much product should be produced, and how much regular and overtime labour hours are required. The objective of the model is to maximize the net revenue less fishing cost, less inventory cost for raw materials and products, less labour cost, and subject to fishing trawler scheduling constraints, quota constraints, processing constraints, and labour allocations.

The remainder of the chapter proceeds as follows. In Section 3.2, we discuss the data used in this study. Section 3.3 presents the model. Section 3.4 presents a sample output of a 10-period model.

3.2 Data used in this study

In this section, we describe the various aspects of the problem solved. We discuss the relevant data for the model, any assumption made, and how the data was obtained.

We collected data from one of the major fisheries in New Zealand. The fishery has two small trawlers and one large trawler. Average expected catch for each small trawler is 12 tonnes per day, and each trawler takes two to three days per fishing trip. The average expected catch for the large trawler is 90 tons per day, and the trawler takes up to 7 to 8 days per trip. The trawlers harvest 8 species over the year. In the running season, the trawlers harvest hoki, roughy, dory, ling, red cod, squid, Barracuda and elephant fish. The company produces 10 different products over the year. The product type varies a little by the fish type. For example, squid can not be used for producing fillet. The fish that cannot be processed during a period remain in inventory and are available for the next period production. The quality of the fish decreases with the increase of inventory days. Similarly, the products that cannot be sold during a period remain in inventory and will be sold in the next period. To allow proper testing of the model, we use some estimated numbers (for example, $FR_{i,l}$ denotes the fraction of different raw materials classified according to their quality).

In the following four subsections, we describe trawler scheduling, processing, and labour allocation.

3.2.1 Trawler scheduling

Suppose a set of quotas for several fish stocks is given, and a set of requirements for raw materials at the processing firm for each period is given. Then a trawler's scheduling problem can be defined as "determine a schedule for each trawler of the stocks to be fished, amount to be caught, and when to be landed, whilst minimizing streaming and catching costs."

At the beginning of a planning horizon, we assign each trawler the home port as the landing port, a set of fish areas where it can go for fishing, and a set of fish stocks which it harvests assuming that the remaining quota in that fishing area is greater than or equal to the amount to be caught. When a trawler reaches its capacity, or when the processing plant needs raw material, then the trawler returns to the port to land its catch. The trawler is then unloaded, cleaned, re-iced, and refuelled. Then it is again available for the next trip. A trawler can make multiple trips over the planning horizon. The trip length for a trawler depends on the capacity of the trawler, demand of the processing plant, and quality of fish. If the amount caught is less than the amount to be caught, then the fishery can use fish from the safety stock.

The trawler operating costs per period include the salary of the crew, diesel cost, and average maintenance of each trawler. These costs vary according to the trawler class. Since the company owns the trawlers, the company pays the crew of the trawlers a salary. Since the trawler operation cost is fixed, we can assume that the landing price which the fishery pays to each trawler for each species and period is zero.

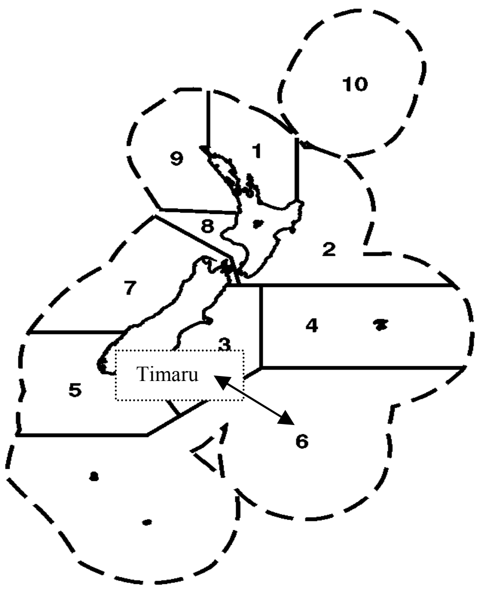

Figure 3.1: Exclusive Economic Zone of New Zealand.

The start and end points of the trawler scheduling in our model are the processing plant which is at the port. We consider that the beginning and the ending ports to be identical. The distances from the processing plant to different fishing areas may differ to a large extent and may impact catch preservation aspects and fuel capacity considerations. A trawler trip is often restricted in length to ensure landing fish of desirable quality and for contractual reasons. On a given trip, a trawler can go to one quota management area at a time and can harvest any number of available fish stocks.

To illustrate a trawler's trip, we show the 10 quota management areas (QMAs) in the 200-mile exclusive economic zone of New Zealand, in Figure 3.1. For example, if a fishery is located at Timaru, then a trip can be defined as the distance travelled from Timaru to the centriod of the fish stock area (say stock area 6), and returning from that stock area to the fishery.

A sample trip of a trawler to a QMA to catch fish from some stocks is presented in Figure 3.2. From the figure, we notice that a trawler goes for fishing to one QMA at a time in a trip and can harvest more than one fish stock.

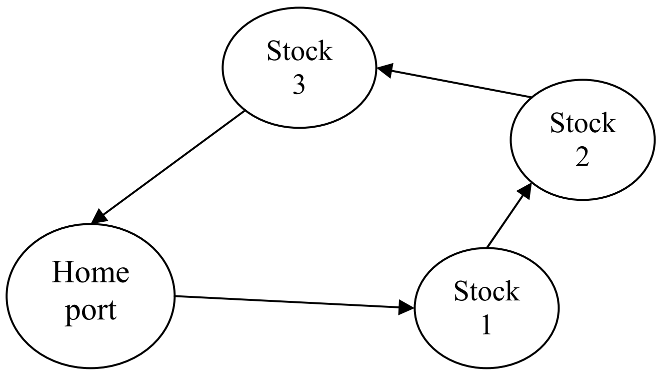

Figure 3.2: A sample trawler's trip

Trawler	Maximum number of periods in a trip	Capacity (Kg)	Operating cost per period ($)
Trawl1	21	85,000	6,000
Trawl2	7	35,000	3,500
Trawl3	7	35,000	3,500

Table 3.1: Sample data for a trawler's trip

In Table 3.1, we present sample data for the maximum number of periods at sea, capacity, and operating cost per period of a trip of trawler. Table 3.2 presents sample average expected catch rate of each trawler over different fish stocks in a given QMA. The fishery was unable to provide sufficient data to allow the reliable determination of catch rate by trawler class, by stock, and by period. Eventually a matrix of average expected catch rates suitable for the purpose of experimenting with the IFPM was generated from the historical landings provided by the fishery and in consultation with the fishery official. The detailed data is presented in Appendix 1.

Trawler	Average expected catch per day (kg)			
	hoki	roughy	dory	ling
Trawl1	6,000	2,500	1,000	2,500
Trawl2	4,000	2,000	1,000	2,000
Trawl3	4,000	2,000	1,000	2,000

Table 3.2: Sample catch rate of each Trawler in a QMA.

In Table 3.3, we present sample data of fish quota owned by the fishery. For quota data, we use quotas allocated to the company for 2005.

Species	Stock	Quota (kg)
hoki	HOK1	2,86,45,910
	HOK10	0

Table 3.3: Sample available fish quota for Hoki.

3.2.2 Processing

We consider the production of fillet (FIL), gutted (GUT), and headed and gutted (HGU) from different fish species. Here we discuss the conversion factors for different product and species.

Conversion Factors: The conversion factor is the amount by which the product weight must be multiplied to determine the "greenweight" equivalent. The "greenweight" is the weight of fish prior to any processing or removal of any part of the fish. For example, 1 kilogram of headed and gutted (HGU) hoki is equivalent to 1.5 kilogram of hoki fish. A sample data of conversion factors for hoki fillet (FIL), gutted (GUT), and headed and gutted (HUG) is presented in Table 3.4. In our model, without loss of generality, we assume that the conversion factors for all the three quality types are identical. For example, to produce 1 kilogram of type 1 fillet the fishery needs 2.65 kilograms of hoki fish. Similarly to produce 1 kilogram of type 2 or type 3 fillet the fishery needs the same amount of raw fish.

Species	Quality Type	Products (kg)		
		Fillet(FIL)	GUT	HGU
Hoki	All	2.65	1.1	1.5

Table 3.4: A sample conversion factor according to product and species.

In Table 3.5, we also present sample selling prices of hoki FIL, GUT, and HGU according to the quality type. For the purpose of executing our model, we use historical records of sales estimates obtained from the company. The sales estimates we assume, are representative of the market demand for fish.

Species	Quality Type	Price of Products ($)		
		FILL	GUT	HGU
hoki	I	2.9	2	2.5
	II	2.5	1.7	2.0
	III	2	1.5	1.5

Table 3.5: A sample selling price according to quality

3.2.3 Labour allocation

We obtained data from the fishery regarding required labour hours per kilogram of product in different work centres for all raw materials and product, the wage rate for regular and overtime labour hours, lower and upper limits of the available labour hours, lower and upper limits of the available overtime labour hours, and the available machine hours. A sample data for the processing time per kilogram of different products is shown in Table 3.6. It seems from the sample data that some of the zero values are identical. This is because, these are not required for these product. For example to produce these products the fishery does not need grinding. But other products may need them.

Products	Labour hour required in each work centre (hour per kg of product)									
	clean	fillet	cut	form	coat	grind	dry	pack	freeze	invt
FIL	.01	.01	0	0	0	0	0	.01	.01	0
GUT	.01	0	0	0.01	0	0	0	.01	.01	0
HGU	.01	0	.01	0	.01	0	0	.01	.01	0

Table 3.6: Required labour per hour per kilogram of product

3.3 Integrated fishery planning model (IFPM)

In this section, we present our mixed integer linear program (MILP) for the co-ordination of trawler scheduling, processing, quota allocation and labour allocation for an integrated commercial fishery. We name the model as integrated fishery planning model (IFPM).

3.3.1 Indices, parameters and decision variables

In this subsection, we define IFPM indices, parameters, and decision variables separately according to trawler scheduling, processing and quota allocation, in Table 3.7 to Table 3.10.

Indices	Descriptions
a	Fishing areas (fish stocks). New Zealand has 10 commercial fish management areas.
c	Work centres, e.g. cleaning, filleting, wrapping, cutting, forming, grinding, drying, packing, and freezing centre.
i	Category of raw material by fish species and size. For example, hoki ≥ 50 cm, hoki ≤ 50 cm, cod, redfish, etc.
j	Type of product, e.g. fillet (FIL), headed and gutted (HGU), and gutted (GUT).
l	Quality of landed fish and product. The classification according to the quality is the same for both raw materials and final products.
p	Factory, e.g. Timaru. In the sample example, we use just one factory.
s, t, u	Time periods. Here u is used as the starting period of a trawler's trip for fishing and t is used as the landing period for trawler scheduling with $u < t$ and s lies between u and t.
v	The trawler. The fishery studied has three trawlers.

Table 3.7: Model Indices

Parameter	Description
A_v	Capacity of trawler *v*.
$BIF_{i,l}$	Beginning inventory of raw fish *i,* of quality *l*.
$C_{a,i,t,v}$	Landing cost that the fishery pays to each vessel *v* of species *i* in stock *a* for each *t*. Since the trawler operation cost is fixed, we assume that the landing price which the fishery pays to each trawler for each species and period is zero.
$E_{a,i,t,v}$	Average expected catch per period *t* for vessel *v* of species *i* from stock *a.* We generate a matrix of average catch rates suitable for the purpose of experimentation of our IFPM in consultation with the company officials.
$ET_{a,u,t,v}$	Amount of fish caught, calculated according to the fishing time by subtracting the travelling and returning time from total time of a trip. *i.e* max (0, min(A_v, $\sum_{s>u}^{t-1}\sum_i E_{a,i,s,v} - \sum_i TR_{a,v} * E_{a,i,u,v} - \sum_i TR_{a,v} * E_{a,i,t-1,v}$)). Here $t - u \leq N_v$.
$FR_{i,l}$	Fraction of the landed raw material *i* according to the quality *l*.
$FRaw_{a,i,v}$	The fraction of the landed raw material *i* from stock *a*, by the trawler *v*.
$Q_{a,i}$	Quota left for species *i* in stock *a* from earlier trading and fishing, i.e $q_{a,i,0}$.
I_t	Cost of holding inventory raw materials during time *t*.
MI	Maximum kilograms capacity of inventory raw materials.
N_v	Maximum number of fishing days for vessel *v* in a planning horizon.
T	Length of planning horizon. The test problem is implemented for a 30-period planning horizon which is divided into several time buckets. The smallest time bucket considered is of 5-period planning horizon.
$TR_{a,v}$	Time required to travel to fish stock *a* for trawlers *v*. Different trawler has different speed. So travelling time to a fishing area is different for different trawler.
$V_{t,v}$	Operating cost for vessel *v* on day *t*.

Table 3.8: Fishing Parameters

Parameter	Description
$BIP_{i,j,l}$	Beginning inventory product of species *i*, product *j,* quality *l*.
$F_{i,j}$	The quantity of raw material *i* required to produce 1 kilogram of product *j*.
$H_{i,j,c,}$	Labour hours required in work centre *c* per kilogram fish *i* in product *j*.
J_t	Inventory holding cost of a product for period *t*.
Lr_t	Labour cost per hour for regular time for period *t*.
Lo	Labour cost per hour for overtime (the overtime labour rate is 25% higher than the regular time labour rate).
LAr_t	Lower bound on regular labour hours on period *t*.
$LR_{i,l,t}$	Lower bound on kilograms of raw material *i* of quality *l* to be processed in period *t*.
MIP	Storage capacity of maximum inventory product.
$P_{i,j,l}$	Profit from product *j* for raw material *i*, of quality *l* (*i.e.,* the weighted net sales price of products *j* for raw material less all variable costs, except labour cost). Estimate of revenues for fresh and frozen products were provided by the company.
Rt	Ratio of overtime labour hours. The overtime labour should not exceed 25% of the regular labour.
UAr_t	Available regular labour hours in period *t*.
$UM_{i,j,l,t}$	Upper bound on kilograms of products *j* of quality *l* sold from raw material *i* for marketing reasons in period *t*.
$UR_{i,l,t}$	Upper bound on kilograms of raw material *i* of quality *l* to be processed in period *t*.

Table 3.9: Processing Parameters

Variable	Description
$f_{a,i,l,t,v}$	Kilograms of fish species *i* of quality *l* from stock *a* landed by trawler *v* in period *t*.
$q_{a,i,t}$	Quota kilograms of species *i* from stock *a* left over as available quota for period *t*.
$r_{i,j,l,t}$	Kilograms of product *j* made from species *i* of quality *l* kept in inventory at the end of planning horizon *t*. We define $r_{i,j,l,0}$ as the initial inventory, a constant.
$s_{i,j,l,t}$	Kilograms of product *j* sold from raw material *i* of quality *l* in period *t*
$w_{p,a,u,t,v}$	1 if trawler *v* steams from the firm *p* to fish stock area *a* on period *u* for fishing and returns in period *t*; 0 otherwise, for all $t = 1 \ldots T$, and all *u:* $1 \leq t\text{-}u \leq N_v$, and all *v*.
$wr_{t,v}$	1 if *v* waits in port during period *t*; 0 otherwise.
$x_{i,j,l,t}$	Kilograms of product *j* produced from raw material *i* of quality *l* in period *t*.
$yo_{t,c}$	Overtime labour hours used in different work centres *c* during period *t*.
yr_c	Regular labour hours used in different work centres *c*. According to the demand of product and supply of raw materials, the model decides the number of regular labour hours. The regular labour cost is the same for all periods.
$z_{i,l,t}$	Kilograms of fish species *i* of quality *l* kept in inventory at the end of planning horizon *t*. We define $z_{i,l,0}$ as the initial inventory, a constant.

Table 3.10: Model decision variables

3.3.2 Objective function

The objective of our IFPM is to maximize total profit, which is revenue from sales, less fishing cost, less production cost, less inventory holding cost. We therefore maximize the expression

$$= \sum_i \sum_j \sum_l \sum_t P_{i,j,l}\, s_{i,j,l,t} - \sum_p \sum_a \sum_u \sum_t \sum_v (t-u) V_{t,v} w_{p,a,u,t,v} - \sum_a \sum_i \sum_l \sum_t \sum_v C_{a,i,t,v}\, f_{a,i,l,t,v}$$
$$- \sum_c \sum_t Lr_t\, yr_c - \sum_c \sum_t Lo\, yo_{t,c} - \sum_i \sum_l \sum_t I_t z_{i,l,t} - \sum_i \sum_j \sum_l \sum_t J_t r_{i,j,l,t}.$$

3.3.3 Constraints

Landed fish constraint: The binary variable $w_{p,a,u,t,v}$ indicates whether a trawler goes fishing or not. If a trawler goes fishing, it will land its fish according to the quality of fish.

$$f_{a,i,l,t,v} = \sum_p \sum_u ET_{a,i,u,t,v} \times FR_{i,l} \times FRaw_{a,i,v} \times w_{p,a,u,t,v} \quad \text{for all } a,\ i,\ l,\ t \text{ and } v. \tag{3.1}$$

Trawler start constraint: A trawler will go fishing or stay at port according to the requirement and profitability of the fishery.

$$\sum_a \sum_p \sum_{t=2}^{N_v} w_{p,a,1,t,v} + wr_{1,v} = 1 \quad \text{for all vessels } v. \tag{3.2}$$

Flow constraint: If a trawler goes fishing, it must come back to land its catch. This constraint also assures that the end of a trawler trip is the beginning of the next trip. If a trawler stays at the port, it will be treated as an idle trip with zero landings.

$$\sum_a \sum_p \sum_{u=1}^{\max\{1,\, t-N_v\}} w_{p,a,u,t,v} + wr_{t-1,v} - wr_{t,v} - \sum_a \sum_p \sum_{t1=t+1}^{\min\{t+N_v,\, T\}} w_{p,a,t,t1,v} = 0 \quad \text{for all t and } v. \tag{3.3}$$

Constraint (3.2) and (3.3) represent the trawler scheduling constraints which are networking constraints.

Bounds on raw materials constraint: The amount of processed raw materials should not exceed the available raw material i of quality l in period t.

$$LR_{i,l,t} \leq \sum_{j} F_{i,j}\ x_{i,j,l,t} \leq UR_{i,l,t} \quad \text{for all } i\text{ , } l \text{ and } t. \tag{3.4}$$

Inventory balance constraint: The fish species i of quality l, which is not used for production in period t, is stored as inventory ($z_{i,l,t}$) for use in the next planning horizon. We define the beginning inventory raw material $z_{i,l,0} = BIF_{i,l}$, a constant. We assume that the caught fish in trawler is not part of inventory.

$$z_{i,l,t-1} + \sum_{a}\sum_{v} f_{a,i,l,t,v} - \sum_{j} F_{i,j}\ x_{i,j,l,t} = z_{i,l,t} \quad \text{for all } i\text{ ,}l\text{, and } t. \tag{3.5}$$

Inventory storage capacity constraint: Inventory of raw material should not exceed maximum storage capacity.

$$\sum_{i}\sum_{l} z_{i,l,t} \leq MI \quad \text{for all } t. \tag{3.6}$$

Safety stock constraint: This model decides how much raw material will be kept as inventory at the end of each period as safety stock.

$$z_{i,l,0} = z_{i,l,T} \qquad \text{for all } i\text{, and } l. \tag{3.6a}$$

Marketing constraint: The amount of product sold depends on the demands in the market. We use the historical records of sales estimates obtained from the company for our IFPM.

$$s_{i,j,l,t} \leq UM_{i,j,l,t} \qquad \text{for all } i\text{ , }j\text{, } l \text{ and } t. \tag{3.7}$$

Inventory products balance constraint: The total inventory of product at the end of last period plus product produced in the current period minus product sold, yields inventory for next period. We define the beginning inventory raw material $r_{i,j,l,0}$ = $BIP_{i,j,l}$, a constant.

$$r_{i,j,l,t-1} + x_{i,j,l,t} - s_{i,j,l,t} = r_{i,j,l,t}. \quad \text{for all } i\text{ , }j\text{, } l \text{ and } t. \tag{3.8}$$

Inventory products storage constraint: The inventory of products during period t should not exceed the maximum storage capacity.

$$\sum_{i}\sum_{j}\sum_{l} r_{i,j,l,t} \le MIP \quad \text{for all } t. \tag{3.9}$$

Labour constraint: The working time required for filleting, packing, freezing, and stocking during period t should not exceed the total available regular time and overtime.

$$\sum_{i}\sum_{j}\sum_{l} H_{i,j,c}\, x_{i,j,l,t} - yr_c - yo_{t,c} \le 0, \quad \text{for all } t, \text{ and } c. \tag{3.10}$$

Available labour constraint: Regular labour hours during period t have lower and upper bounds.

$$LAr_t \le yr_c \le UAr_t \quad \text{for all } t, \text{ and } c. \tag{3.11}$$

Overtime labour constraint: Overtime labour should not exceed a fraction R_t (25%) of regular amount of labour.

$$yo_{t,c} \le Rt \times yr_c \quad \text{for all } t, \text{ and } c. \tag{3.12}$$

Quota constraint: The available quota must balance over time. Hence, last period's quota minus the fish landed gives the remaining quota for the next period. We set the initial quota $q_{a,i,0} = Q_{a,i}$. The quota is reduced by the amount caught during a period. In our model, we use the quota smoothly over the planning horizon.

$$q_{a,i,t-1} - \sum_{l}\sum_{v} f_{a,i,l,t,v} = q_{a,i,t} \quad \text{for all } a, i \text{ and } t. \tag{3.13}$$

Non-negativity:

$$x_{i,j,l,t},\ s_{i,j,l,t},\ r_{i,j,l,t},\ yr_c,\ yo_{t,c},\ f_{a,i,l,t,v},\ w_{p,a,u,t,v},\ wr_{t,v},\ z_{i,l,t},\ q_{a,i,t} \ge 0 \tag{3.14a}$$

Binary variables:

$$w_{p,a,u,t,v} \in \{0,1\},\ wr_{t,v} \in \{0,1\} \tag{3.14b}$$

3.4 Sample output from a 10-period model

In this section, we present an example solution of a 10-period model. Because a 30-period problem results in a large number of variables and constraints, we solve the IFPM by reducing the size of the problem for easier explanation. We discussed the relevant data for the model, any assumption made, and how the data was obtained in the Section 3.2. The parameters such as catch rate, production revenue, inventory costs, etc., are averaged over the planning horizon. The detailed data is presented in Appendix 1.

In Figure 3.3, we observed that, in its first trip, trawler 1 goes for fishing to area 3 on period 1 and lands its catch (84,150 kg.) on period 4. Trawler 2 goes for fishing to area 3 on period 1 and lands its catch (34,650 kg.) on period 3. So does trawler 3. The details of trawler scheduling and amount of fish landed are shown in Figure 3.3 and Table 3.11.

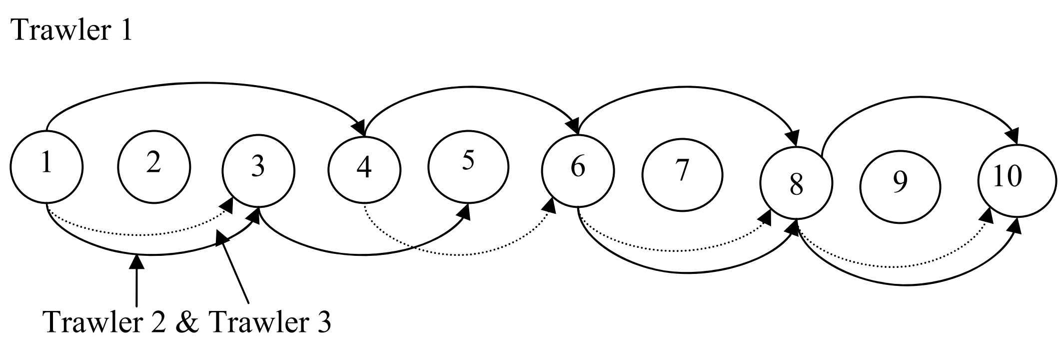

Figure 3.3: A sample fishing trawler scheduling for a 10-period model.

Trawler	Period	Amount of fish landed
Trawler 2	3	34,650
Trawler 3	3	34,650
Trawler 1	4	84,150
Trawler 2	5	34,650
Trawler 1	6	70,785
Trawler 3	6	34,650
Trawler 1	8	70,785
Trawler 2	8	34,650
Trawler 3	8	34,650
Trawler 1	10	70,785
Trawler 2	10	34,650
Trawler 3	10	34,650

Table 3.11: Amount of fish landed in different periods in a 10-period model.

In Table 3.12, we present the amount of type 1 fish of each species landed by trawler 1 from quota management area 3. Since on period 4 a 3-day trip was landed, the amounts of landed fish of each species were higher than the other trips which were 2-day trips. The detailed result is presented in Appendix 1.

Landed period	Species (Kg.)						
	Barracuta	Dory	Efish	Ling	Redcod	Roughy	Squid
4	5,890.5	9,817.5	3,64.6	1,96.4	1,936.5	5,890.5	3,927.0
6	4,954.9	8,258.2	3,06.7	1,65.2	1,651.6	4,954.9	3,303.3
8	4,954.9	8,258.2	3,06.7	1,65.2	1,651.6	4,954.9	3,303.3
10	4,954.9	8,258.2	3,06.7	1,65.2	1,651.6	4,954.9	3,303.3

Table 3.12: Sample output of the amount of type 1 fish of each species landed by trawler 1 from area 3.

In Table 3.13, we present a sample output of the amount of type 1 fish of each species stored as inventory raw materials in different period. We noticed that the beginning and the final inventory of each raw material is equal.

Period	Species (Kg.)					
	Barracuda	Dory	Efish	Redcod	Roughy	Squid
0	8,626	16,670.0	722.5	2,574.7	3,828	5,700
1	0	10,520.0	722.5	2,500.0	0	2,850
3	0	2,267.7	0	0	0	0
4	0	8,055.0	293.9	0	0	0
5	0	1,089.0	0	0	0	0
6	0	10,181.0	0	0	0	0
8	0	14,412.9	0	2,092.8	1,766.6	0
9	0	8,394.0	0	0	0	
10	8,626	16,670.0	722.5	2,574.7	3,828.0	5,700

Table 3.13: Sample output of the amount of type 1 raw fish of each species stored.

In Table 3.14, we present the amount of type 1 fillet of each species produced in different period. Fillet cannot be produced from squid.

Period	Fillet (FIL) (Kg.)						
	Barracuda	Dory	Efish	Ling	Redcod	Roughy	Squid
1	3,750.5	1,500.0	0	0	29.9	1093.8	-
2	0	1,500.0	253.5	0	1,000.0	0	-
3	2,681.6	1,500.0	145.9	16.5	822.4	1,178.0	-
4	2,561.0	1,500.0	127.9	70.1	667.8	1,683.0	-
5	1,340.8	3,000.0	72.9	8.3	528.7	589.0	-
6	3,495.0	0	180.6	67.0	234.7	1,500.0	-
7	0	3,050.5	0	0	837.0	504.7	-
8	4,835.0	0	253.5	75.5	1,000.0	1,500.0	-
9	0	1,449.5	0	0	483.0	1,093.8	-
10	1,085.0	1,500.0	0	75.5	453.0	1,500.0	-

Table 3.14: Sample output of the amount of type 1 fillet (FIL) produced.

In Table 3.15, we present the amount of type 1 fillet of each species sold in different periods.

Period	Fillet (FIL) (Kg.)						
	Barracuda	Dory	Efish	Ling	Redcod	Roughy	Squid
1	2,000	1,500	0	0	29.9	1,093.8	-
2	1,750	1,500	253.5	0	1,000	0	-
3	2,000	1,500	145.9	16.5	822.4	1178	-
4	2,000	1,500	127.9	70.1	667.8	1500	-
5	2,000	1,500	72.9	8.3	528.7	772	-
6	2,000	1,500	180.6	67	234.7	1,500	-
7	2,000	1,500	0	0	837	504.7	-
8	2,000	1,500	253.5	75.5	1,000	1,500	-
9	2,000	1,500	0	0	483	1,093.8	-
10	2,000	1,500	0	75.5	453	1,500	-

Table 3.15: Sample output of the amount of type 1 fillet of each species sold.

In Table 3.16, we present the amount of type 1 fillet of each species stored as inventory products in different periods. We observed that only barracuda fillets, dory fillets and roughy fillets were stored because the landed fish of these species were higher than the other species and as a result the processed amount of these species were high.

Period	Fillet (FIL) (Kg.)		
	Barracuda	Dory	Roughy
1	1,750.5	0	0
3	681.6	0	0
4	1,242.7	0	183
5	583.5	1,500	0
6	2,078.6	0	0
7	78.63	1,550.5	0
8	2,914.5	50.5	0
9	914.5	0	0

Table 3.16: Sample output of the amount of type 1 fillet of each species stored.

Sample output in SVG

In this section, we present a partial sample output from a 5-period model in scalable vector graphics (SVG). SVG is a web-standard for describing two dimensional vector graphics which integrates vector graphics, raster graphics and text. Adobe, amongst others, provides a free SVG plug-in for Internet Explorer. It therefore costs nothing for experimenting with this technology.

Using SVG, we create simple graphics in just a few lines of text and displayed much information in one web page in Figure 3.4. It shows the amount of fish landed by each trawler in period 3, the amount of each quality product produced in period 3, and the amount of raw material kept as inventory at the end of period 3 which will be available during the next period.

Period	Fish type	Quality type	Area	Trawler	landed fish	Products	produced	inventory fish
Day 3	roughy	type1	area3	tr1	4954.95	FILL	1500	3828.3
				tr2	2061.68	FILL	1500	
				tr3	2061.68	FILL	1500	
		type2	area3	tr1	4954.95	FILL	1500	3828.3
				tr2	2061.68	FILL	1500	
				tr3	2061.68	FILL	1500	
		type3	area3	tr1	4954.95	FILL	1500	3828.3
				tr2	2061.68	FILL	1500	
				tr3	2061.68	FILL	1500	
	dory	type1	area3	tr1	8258.25	FILL	1500	10525.9
				tr2	3083.85	FILL	1500	
				tr3	3083.85	FILL	1500	
		type2	area3	tr1	8258.25	FILL	1500	10525.9
				tr2	3083.85	FILL	1500	
				tr3	3083.85	FILL	1500	
		type3	area3	tr1	8258.25	FILL	1500	10525.9
				tr2	3083.85	FILL	1500	
				tr3	3083.85	FILL	1500	
	ling	type1	area3	tr1	165.165	FILL	75.4875	0
				tr2	23.1	FILL	75.4875	
				tr3	23.1	FILL	75.4875	
		type2	area3	tr1	165.165	FILL	75.4875	0
				tr2	23.1	FILL	75.4875	
				tr3	23.1	FILL	75.4875	
		type3	area3	tr1	165.165	FILL	75.4875	0
				tr2	23.1	FILL	75.4875	
				tr3	23.1	FILL	75.4875	
	redcod	type1	area3	tr1	1651.65	FILL	46.1749	3592.11
				tr2	1027.95	FILL	46.1749	
				tr3	1027.95	FILL	46.1749	
	squid	type1	area3	tr1	3303.3	GUT	446.889	3973.35
						HGU	1500	
				tr2	2061.68	GUT	446.889	
						HGU	1500	
				tr3	2061.68	GUT	446.889	
						HGU	1500	
		type2	area3	tr1	3303.3	GUT	446.889	3973.35
						HGU	1500	
				tr2	2061.68	GUT	446.889	
						HGU	1500	
				tr3	2061.68	GUT	446.889	
						HGU	1500	
		type3	area3	tr1	3303.3	GUT	1279	2850
						HGU	1500	
				tr2	2061.68	GUT	1279	
						HGU	1500	
				tr3	2061.68	GUT	1279	
						HGU	1500	
	bcuda	type1	area3	tr1	4954.95	FILL	4835.93	0
				tr2	3083.85	FILL	4835.93	
				tr3	3083.85	FILL	4835.93	
		type2	area3	tr1	4954.95	FILL	4835.93	0
				tr2	3083.85	FILL	4835.93	
				tr3	3083.85	FILL	4835.93	
		type3	area3	tr1	4954.95	FILL	4835.93	0
				tr2	3083.85	FILL	4835.93	
				tr3	3083.85	FILL	4835.93	

Figure 3.4: SVG output of period 3 of a 5-period model. It shows the type and amount of fish landed by each trawler from a fishing area, amount of product produced and amount of raw fish kept as inventory.

3.5 Conclusion

We developed an integrated fishery planning model (IFPM) to schedule fishing trawlers and to plan production for an integrated commercial fishery. The model co-ordinates trawler scheduling, fishing, catch quota allocations, processing and labour allocation of fisheries. We hope that, given the complexity of the fishery problem and the level of uncertainty in the catch rate, the IFPM will provide an efficient approach to address the decision making to be made by the fishery.

To study the behaviour of the IFPM in detail, a sensitivity analysis will be performed in Chapter 4, so the fishery can develop guidelines for updating data and decision plans in the light of new information obtained from this sensitivity analysis.

The uncontrollable factors such as variability in catch rate, weather conditions, available quotas, and seasonality of fish stocks' availability make the integrated fishery planning decisions difficult. Ways to manage the uncertainties of the integrated fishery are presented in Chapter 5.

For a real-world problem, the IFPM is a difficult to solve MILP, with trawler scheduling and processing scheduling connected by side constraints. Without a special algorithm, IFPM takes a long time to solve. For example, a 29 or 30-period model takes more than five hours to solve. We will try relaxation, decomposition and pricing methods to decouple trawler scheduling and processing for solving the IFPM in Chapter 6, 7 and 8.

Chapter 4

Analysis of the Model

4.1 Introduction

In this chapter, we discuss the computational time and structure of the model. We apply a traditional decomposition method of rolling horizon for reducing the size of the problem for solving the IFPM.

Unlike popular problems in the OR literature, standard test problems for our model do not exist. Modifying one real-model data set, we generate three more different problem instances. Care however must be exercised in order to generate instances with feasible solutions.

We also propose a simple safety stock approach to deal with the end-of-planning horizon effect. We then investigate a modification of the trawler scheduling to allow that the trawlers do not necessarily need to be at the port at the beginning and end of a planning horizon.

The remainder of this chapter is organized as follows. Section 4.2 presents the structure of the model, computation time and number of variables in different planning horizon models. In Section 4.3, we present the rolling horizon approach. In Section 4.4, we analyse the impact of quota allocations on the profit and also compare the average profit of different planning horizon models. In Section 4.5, we present the safety stock approach. In Section 4.6, we modify the trawler scheduling to allow the trawlers to continue its next trip. And in Section 4.7, we develop some more problem instances to observe the workability of the IFPM.

4.2 Structure of the IFPM and computation times

In this section, we present the structure of the IFPM and discuss the computational difficulty for solving the longer planning horizon models.

4.2.1 Structure of the model (IFPM)

The IFPM consists of a trawler scheduling and a processing subproblem along with complicating side constraints containing variables from both the subproblems. And hence, it is hard to solve.

Here we present the structure of the model in matrix-vector notation.

Parameters

c^1, c^2, c^3, unit profit of trawler operation, raw fish inventory, and fish processing, respectively,

A^0, quantity of fish landed per trip in each period,

D^1, mass balance coefficients on each trawler in each period,

D^2, mass balance coefficients on fish within the processing factory,

A^1, A^2, mass balance coefficients governing transformation of raw fish into finished product.

Decision variables

w, binary variables indicating whether a trawler takes a given trip,

f, raw fish inventory, indicating the current quantity of each type of raw fish in each period,

x, fish processing variables, indicating that a given type of raw fish is converted into a given product.

IFPM: maximize $c^1 w + c^2 f + c^3 x$,

subject to

Inventory supply constraints,	$A^0 w$	$+ f$		$= 0.$	(4.1)
Trawler scheduling constraints,	$D^1 w$			$= b^1.$	(4.2)
Processing constraints			$D^2 x$	$= b^2.$	(4.3)
Inventory demand constraints,		$A^1 f$	$+ A^2 x$	$= b^0.$	(4.4)
	$w \in \{0,1\}$				(4.5a)
		$f,\ x \geq 0.$			(4.5b)

Equation (4.1) represents the relationship of the trawler scheduling variables w to landed fish f, as a mass balance in movement of fish from trawlers to the factory. Equation (4.2) expresses the constraints involving only trawler scheduling, indicating, for example, that a trawler may be in only one place at a time. Equation (4.3) expresses fish processing constraints, modelling the flow of fish through the factory as raw fish is made into various products. Equation (4.4) represents the mass balance constraints, representing the flow of raw landed fish inventory into the fish processing factory.

4.2.2 Test problems generation

In Section 3.2 of Chapter 3, we discussed the data used for testing the integrated fishery planning model (IFPM). In this section, modifying the original problem data, we extract three more different test problems. These three problems are referred to as "IFPMS," "IFPML," and "IFPMXL". These problems are different in many aspects. For example, the "IFPMS" is smaller than the original problem. It has fewer trawlers and quality types. We use two quality types of raw fish and products; acceptable and unacceptable. The unacceptable raw materials are used to produce fish meal. The "IFPML" is larger than the original problem. It has a higher number of trawler and stock areas. The "IFPMXL" has a higher number of trawlers and stock areas than the other problems. A summary of these problems for the IFPM is given in the Table 4.1.

Characteristics		Original Problem	IFPMS	IFPML	IFPMXL
Number of trawlers	:	3	2	4	6
Number of factories	:	1	1	1	1
Number of species	:	8	8	8	8
Number of stock areas	:	2	2	3	4
Number of quality types	:	3	2	3	3
Number of product types	:	3	4	3	3
Number of constraints	:	10,885	6,550	15,456	24,404
Number of continuous variables	:	9,685	5,785	11,533	12,981
Number of integer variables	:	2,556	1728	5,124	7,620

Table 4.1: A summary of four different problems of 30-period planning horizons for the IFPM.

In the following section, we discuss the solution time and difficulties for solving longer planning horizon problems.

4.2.3 Computation times

We implemented our model using the AMPL modelling language (Fourer *et al*., 1993) and used CPLEX (ILOG Corp., www.ilog.com) to solve it. Varying the number of periods of the planning horizon from 5 to 30, we solved our model on computer with an Intel Pentium III processor with a clock speed of 665 MHz and 384 MB of RAM. Table 4.2 shows the optimal profit, computation time, number of integer and continuous variables associated with each planning horizon of 5 to 30-periods of the original problem. AMPL's presolve eliminates some variables. For example, a 30-period model has 14,699 variables. But presolve eliminates 2,458 variables and shows a total of 12,241 variables (2,556 integer and 9,685 continuous). The longer planning horizon models take considerably longer time to solve. For example, we ran and abandoned a 29 and a 30-period model after more than five hours.

Planning Horizon	Solution time (sec)	$v(P)$ (\$)	$v(\bar{P})$ (\$)	Variables	
				Integer	Continuous
5	3	522,764	522,764	156	4,110
6	4	556,945	557,440	180	4,333
7	3	770,767	770,767	216	4,556
8	7	812,587	813,076	258	4,779
9	4	1,013,345	1,013,345	306	5,002
10	10	1,065,775	1,066,350	360	5,225
11	6	1,255,777	1,255,777	414	5,448
12	13	1,313,945	1,314,621	468	5,671
13	85	1,431,831	1,466,321	522	5894
14	28	1,506,253	1,515,077	576	6,117
15	53	1,582,008	1,607,944	630	6,340
16	60	1,621743	1,648,103	684	6,563
17	73	1,695,835	1,734,379	738	6,786
18	356	1,746,724	1,774,867	792	7,009
19	81	1,826,217	1,859,060	846	7,232
20	131	1,880,196	1,898,411	900	7,455
21	166	1,931,858	1,963,397	954	7,678
22	1354	1,962,473	1,992,527	1,008	9,701
23	1429	2,007,252	2,056,248	1,058	8,124
24	1632	2,048,128	2,084,239	1,740	8,347
25	153	2,121,887	2,141,757	1,872	8,570
26	328	2,146,273	2,173,053	2,000	8,793
27	1008	2,192,681	2,220,159	2,136	9,016
28	331	2,236,589	2,258,272	2,274	9,239
*29	16,745	2,261,176	2,295,345	2,414	9,462
*30	18,240	2,300,871	2,331,036	2,556	9,685

Table 4.2. IP profit, number of integer and continuous variables obtained from the solution of 5 to 30-period original problem.

* The solution process was abandoned after more than five hours, and so the solution shown may not be optimal.

We also tried to run the 30-period model by a computer with 1.73MHz Pentium M, with 512 MB of RAM. But we gave up after 28 hours. Windows indicated that it had run out of memory, and was trying to allocate more virtual (hard disk) memory. Thus, we can say for sure that this model would require a lot of memory.

Using the same computer we tried to solve the problems IFPMS, IFPML and IFPMXL for 5 to 30 period planning horizons. The results are shown in Table 4.3.

Problem	PH	Solution time (Seconds)	IP profit ($)	LP profit ($)	Variables	
					Integer	Continuous
IFPMS	5	02	335,477	335,477	50	910
	10	03	701,182	702,866	200	1,885
	15	03	1,123,295	1,123,295	450	2,860
	20	84	1,439,023	1,461,530	800	3,835
	25	324	1,705,280	1,733,364	1,238	4,810
	*30	1930	1,874,130	1,905,126	1,728	5,785
IFPML	5	03	660,701	665,741	140	1,683
	10	15	1,347,194	1,353,447	580	3,653
	15	359	1,852,260	1,891,241	1,320	5,623
	*20	2034	2,196,291	2,234,440	2,360	7,593
	*25	2152	2,400,920	2,444,345	3,664	9,563
	*30	2213	2,550,260	2,605,895	5,124	11,533
IFPMXL	5	03	732,706	747,420	210	1,806
	10	34	1,542,810	1,554,154	810	4,041
	15	36,540	1,994,834	2,006,230	1,932	6,276
	*20	2190	2,248,057	2,262,451	3,522	8,511
	*25	2345	2,396,554	2,416,450	5,490	10,746
	*30	2590	2,546,817	2,568,376	7,620	12,981

Table 4.3: IP profit, number of integer and continuous variables obtained from the solution of 5 to 30-period IFPMS, IFPML, and IFPMXL.

* The solution process was abandoned after more than five hours, and so the solution shown may not be optimal.

We tried and abandoned a 30-period model of IFPMS after more than 12 hours. To solve a 20-period model of IFPML, we tried more than 5 hours and abandoned. We also attempted to solve 25 and 30-period models of IFPML but failed to solve. A 15-period model of IFPMXL took 10 hours and 9 minutes to solve and yielded a total profit of $1,994,830. We also attempted to solve 20, 25 and 30-period models but these failed to solve.

Therefore, from the solution times of all four different problem instances, we found that the longer planning horizon problems are hard to solve. To help the fishery to solve the IFPM efficiently, we will apply a rolling horizon approach in the following section.

4.3 Rolling horizon approach

In this section, we examine the value of using a rolling horizon approach for planning and implementing fishery plans. There are several reasons behind the use of this rolling horizon approach. First, to reduce the size of the problem to make it solvable. Second, to overcome the difficulty of catch and demand forecasts for a long horizon, the manager of the fishery may use a shorter planning horizon. And third, to deal with the end-of planning horizon effect.

The procedure of updating forecasts and solving the problem periodically is referred to as a rolling horizon approach. A rolling horizon approach (Blackburn & Millen (1980), Wagner & Whitin (1958)) is a strategy for decomposing a large problem to make it solvable, and for managing the end-of planning horizon effect in deterministic models. This approach has been widely used in production planning (Fisher et al, (2001)). Millar (1998) analyzed the impact of rolling horizon planning on the cost of industrial fishing activity. He analyzed the rolling horizon planning for a MILP model which addressed only the fishing trawler scheduling of an integrated fishery.

Also, due partly to the inability to solve large MILP models and partly to inability to forecast catch and demand, the planning horizon is necessarily short. To overcome the difficulty of catch and demand forecasts for a long horizon, managers may use a shorter planning horizon than any reasonable estimate of the firm’s real future horizon. This results in end-of-planning-horizon effects, which are suboptimal solutions. For example, deterministic MILP models tend to leave zero inventories in the final period unless a minimum final inventory is prescribed. Because, if there is no need for the inventory holdings, the fishery will be interested to process all of the landed fish as soon as it is available and sell the products for profit. Also why pay inventory holding costs of raw fish if there is no need for final inventory.

In the following section, we present the rolling horizon algorithm for the IFPM along with numerical illustrations.

4.3.1 Rolling horizon algorithm

In a rolling horizon, we want to solve for a planning horizon T. We will solve a set of models each with horizon $T2$ where $T2 \ll T$. We initialize the starting period $T1$ of the planning horizon to 1. We will fix and implement decisions and data for fixed horizon η where $\eta \ll T2$. The number of models with horizon $T2$ = round$\left(\frac{T}{\eta}\right)-1$.

We then present the rolling horizon algorithm as follows.

Step 1. Solve each model with horizon $T2$ for periods $T1$, $T1 + 1$, $T1 + 2, \ldots, T2-1$.

Step 2. Fix and implement decisions and data for $T1$ to $T1 + \eta - 1$.

Step 3. Set $T1 = T1 + \eta$ and $T2 = T2 + \eta$.

Step 4. If $T2 < T$, go back to Step 1. Else stop.

To gain insight into the effectiveness of the rolling horizon approach as a mechanism to decompose the model and to deal with the end-of-planning horizon effect, we investigate the relationship between rolling and planning horizons and the total profit of the fishery.

For example, to solve a $T = 30$-period planning horizon model, we ran $T2 = 10$-period models by fixing and implementing the decisions and data for fixed horizon

η = 5-periods, since the first five periods are more certain and the last five periods are less certain, and also because of the difficulty in solving longer planning horizon models. Figure 4.1 shows the rolling horizon *T*2 and fixed horizons η of a T = 30-period planning horizon. Results for different planning horizon models are shown in Table 4.4.

		Profit in planning horizon T ($\$ \times 10^6$)							
*T*2	η	10	12	15	18	20	24	25	30
10	5	-	-	$1.51	-	$1.70	-	$1.79	$1.85
12	6	-	-	-	$1.61	-	$1.79	-	$1.88
	Optimum	$1.06	$1.31	$1.58	$1.75	$1.88	$1.98	$2.12	$2.30

Table 4.4: Profit of different planning horizons for different rolling and fixed horizon.

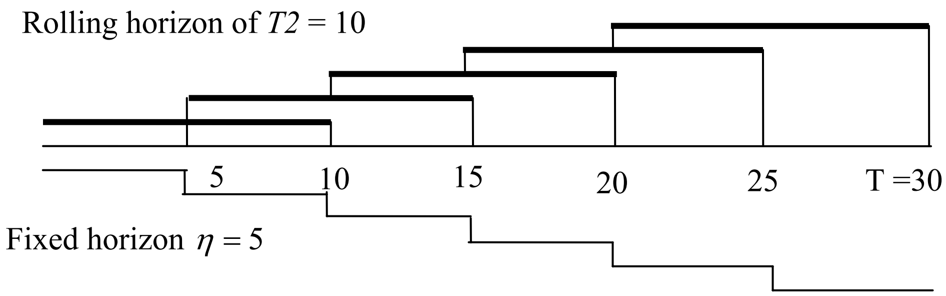

Figure 4.1: Fixed and rolling horizon for a 30-period planning horizon.

Keeping parameters unchanged, we solved the same 30-period model by rolling a T*2* = 12-period model by fixing every η = 6 periods. Results are shown in Table 4.4. A 12-period model fixed every six period yields a better solution than a rolling horizon of 10-period, because, at the first 12-period solution, the model allocates labour for a longer horizon than the 10-period, and so the idle time at the latter part of the 30-period planning horizon was less than that of the10-period rolling horizon.

Also at the beginning of the rolling horizon, the fishery has a lot of fish quota, so the initial model used more labour for fish processing. Since we fixed this labour for the later horizons which got fewer quotas, the later periods yielded less profit and higher idle time, which creates the end-of-planning horizon effect. To cope with this, the fishery needs to pay more attention to labour allocation. This can be done by setting the average amount of labour used in the entire horizon. For this, the fishery could calculate the total quota available for the entire horizon and calculate the approximate labour hour require for processing per kilogram of fish quota. Suppose the total available quota is Q kilograms and the required labour time for processing a kilogram of raw fish is approximately h hours (averaged over all fish and quality types). Therefore, the required time for processing Q kilograms of raw fish is $Q \times h$ hours. Hence, for a T periods planning horizon in each period the fishery is required $\frac{Q \times h}{T}$ hours of labour time. However, we can not be sure that the entire quota Q will be used in this horizon.

Instead, the fishery can allocate labour in another way. In this way, the rolling horizon model needs to be solved twice. At the first time, the model will allocate the labour hours for the first rolling model, and then fix the regular labour for the models of the later parts of the rolling horizon. We observe the actual labour used both regular and overtime, and also idle times. Based on the actual labour used, the fishery can set the average labour per period for the entire horizon.

For example, we first ran a T= 30-period planning horizon of the original problem with $T2$ = 10 and η = 5, which yielded a profit of \$1,859,278. The total labour actually used was 32650 hours. We then set the average labour time (32650 / 30) =

1085 for each period of the entire planning horizon; in the second run, it yielded a total profit of $2,087,490 with a 12.3% increase in the profit, but still 9% less than the direct solution of a 30-period problem.

We do the same experiments with IFPMS, IFPML, and IFPMXL. The results are shown in Table 4.5. From these problems, we notice that these solutions are still 11% to 16% less than the direct solution profit of 30-period models.

Problem	T	$T2$	μ	Direct solution profit ($)	Profit in 1st run ($)	Profit in 2nd run ($)	% change in profit
IFPMS	30	10	5	1,874,130	1,436,371	1,551,080	+7.9
IFPML	30	10	5	2,550,260	1,849,005	2,266,360	+22.6
IFPMXL	30	10	5	2,546,817	1,717,530	2,358,950	+37.3

Table 4.5: 30-period planning horizon with 10-period rolling horizon for IFPMS, IFPML, and IFPMXL.

In this section, we examined a rolling horizon approach to deal with the large problem size, catch data forecasting, and the end-of planning-horizon effect. We found from the solutions of the four different problem instances that the rolling horizon approach was about 9% to 16% far from the optimum. The smooth allocation of labour improves the profit slightly. Therefore, planning for overly short planning horizons can be detrimental to the profitability of the firm.

Rather than allocating labour by period, we could choose to allocate available quota, as we will do in the next section.

4.4 Smooth allocation of quotas

In this section, we analyse the impact of quota allocations on the total profit of the fishery. We also discuss the impact of quotas on the length of planning horizons.

In this experiment, we increase the available quotas of each species and stock area by 10% at a time up to 40% and solve the IFPM for different planning horizon models. When the available quota is increased, we expect the number of trips will increase, since the trawler will get more fish to catch and as a result the landed fish will be increased. Consequently the profit of the fishery will be increased.

We find that the available quota is increased, the number of trawlers' trip taken, increases and as a result the amount of landings and total profit of the fishery increases.

Changing the available quota, we solve the three other different problems IFPMS, IFPML, and IFPMXL. The results were consistent with that of the original problem.

Changing the planning horizon length and keeping other parameters constant, we solved 5, 10, 15, 20, 25, and 30-period models of the original problem, and noticed that a longer planning horizon had a lower average profit per period. Since the fish quota is reduced by the amount of raw fish caught, the longer planning horizon models obtains lower average quota per period. If the available quota of a fish species finishes during a trawler's trip, then the trawler comes back even if it is not full.

The average profit per period of a 5, 10, or 15-period model is similar. That is, a 10-period model approximately yields a profit of two 5-period models, and a 15-period model yields a profit of three 5-period model. But a 20, 25, or 30-period model does not yield profits of 4, 5, or 6 times the 5-period model. Similarly a 20 or 30-period model does not yield a profit of twice or thrice a 10-period model, because for the longer planning horizon the model gets less average quotas and so the model uses fewer regular labour hours for each period and as a result the fishery processes less product resulting in less profit as we will see in the following experiment.

We solve the first 10-period horizon model which yields a total profit of $1,065,775 and uses 1459 hours of regular labour per period. We reduce the quota by the amount of fish caught during this horizon but allow the model to decide the amount of regular labour to be used per period. We also fix the initial quota obtained from the first 10-period model. We observe that the 2nd and 3rd 10-period model uses 901 hours and 586 hours of regular labour per period with no idle hours and yields total profit of $636,262 and $281,318 respectively. The total profit from these three 10-period models is $1,983,355 where as a direct 30-period model yields a total profit of $2,300,871. We also notice that the three 10-period models use 12, 11 and 7 trips respectively which in total are 30 trips. A direct 30-period model use 35-period trips. Results are shown in Table 4.6. Similarly, we do the same experiment with two 15-period models one after another and observe that the 2nd 15-period horizon model yields lower profit and produces idle time. From these experiments, we found that the quota allocation is important. So we observe the effect of smooth quota allocation on the profit in the following experiment.

Planning horizon	Profit ($)	Number of trawler trips	Amount of fish landed (kg)
1st 10-period	1,065,775	12	573,705
2nd 10-period	636,263	11	451,440
3rd 10-period	281,318	7	320,512
Total of three 10-periods	1,983,356	30	1,354,657
30-period (direct)	2,300,871	35	1,530,540

Table 4.6: Comparison of three 10-period horizons to a 30-period horizon.

In this experiment, we allocated one-third of the total available quota for a 30-period horizon to each of the three 10-period horizons. Each 10-period model yielded a profit of $724,007. So, three of these planning horizons yielded an average profit of $724,007 per period which is close to the average profit of a direct 30-period model

($76,696 per period). The total profit from these three 10-period models is $2,172,021 which is the closest profit to a 30-period direct solution profit ($2,300,871) resulting in 5.6% lower profit than the direct solution of a 30-period model.

So we conclude that, the fishery could reduce the solution gap by smoothing allocation of available fish quota with a rolling horizon approach but the result is still about 5% far from the optimum. We also observed that, smoothing the quota allocation results in higher profit than that of smoothing labour.

4.5 Safety stock approach

In this section, we propose a simple safety stock approach to deal with the end-of-planning-horizon effects. This tool gives management a way to calculate a profit maximizing safety stock that deals with the man-made variability due to the trawler scheduling. Safety stock has been widely used to overcome these problems, especially in material requirement planning systems (Wagner & Whitin (1958), Blackburn & Millen (1980), Fisher et al, (2001)).

We set a constraint which ensures that the beginning inventory of raw materials equals the final inventory. Because, in real life, the fishery needs initial and final inventories but they do not have to be the same. At the beginning of a planning horizon, the trawlers need at least two days for fishing and landing their catch. If there is no initial inventory, there will be no activities in the processing plant and all labour will have to sit idly during these two periods. Again at the end of a planning horizon, if there is no final inventory then the next planning horizon will face the same problem. On the other hand, if the fishery has a sufficiently large amount of initial inventory on hand, there will be no trawler scheduling needed for processing to begin.

So the fishery needs an appropriate amount of initial and final inventory as safety stock. We set that the initial inventory raw material equals the final inventory raw material. This is ensured by the constraint here. This approach can be considered as one of many alternatives.

The constraint takes the following form

$$z_{i,l,0} = z_{i,l,T} \qquad \text{for all } i, \text{ and } l. \tag{4.6}$$

The model decides how much raw material will be kept as inventory at the end of each period as safety stock. This type of safety stock protects against variability created by the trawler schedule (man-made variability). This safety stock balances the inventory holding cost, the idle time and overtime.

For example, we solve a 10-period model of the original problem which yielded a total profit of $1,065,775 which is higher than all solutions by naively set inventory (see Table 4.7). We observe that the solution has a total of 104,481 kg of beginning and final inventory raw materials as safety stock.

Beginning inventory fish	Profit ($)	Regular labour (hour)	Overtime hour (hour)	Idle labour (hour)
0	914,129	1,705	1,479	3,410
25,950	935,265	1,638	1,616	2,910
50,100	959,028	1,501	1,330	1,781
1,50,000	1,034,318	1,371	1,382	429
Safety stock approach	1,065,775	1,459	0	0

Table 4.7: Comparison of effect of beginning inventory on profit and labour.

Comparing the safety stock approach and naively set inventory, we conclude that the safety stock approach results in higher profit than that of naively set inventory. But this high inventory size also impacts on profit.

We performed some more experiments with three other different examples (IFPMS, IFPML and IFPMXL). The results are consistent with that of the original problem.

Since we assumed that the initial and final position of the trawlers will be in the port, as a strategy, we observe that at the beginning of the planning horizon the factory has to wait for the first landings for at least two days. As a result of this assumption the model requires a higher level of initial inventory raw materials as safety stock. The short planning horizon models (for example a 10-period model) face this problem too often. The solution is "biased," because, within this short planning horizon the trawlers are restricted to be in the port at the beginning and end periods of that planning horizon. To reduce the impact of this, we will modify the trawler scheduling in the following section.

4.6 Continuous trawler scheduling

In the previous section, we assume that at the beginning and end of a planning horizon, trawlers must be at the port. As a result the processing factory has to wait for the first landing for at least two days. But in real-world trawler scheduling is a continuous process. At the beginning of a planning horizon some trawlers may be already engaged in fishing for some days, some trawlers may be landing their catch, etc. In order to allow this in the IFPM, we allow the trawlers to go for fishing as a continuous process. i.e., we allow cyclic trawler scheduling.

For this we create a "set trips" as follows.

trips:= {*a* in stocks, *u* in 1..*T*, t in 1..*T*, *v* in trawler:($u+1<t$ or $(T+t)-u \le N_V$) and $(T+t)-u>1$ and $t-u \le N_v$ }.

We then change the expected catch parameter ET as follows. Amount of fish caught, calculated according to the fishing time by subtracting the travelling and returning time from total time of a trip. *i.e.,*

$$ET_{a,u,t,v} = \max\left(0, \min\left(\begin{array}{l} A_v, \quad if\ t > u\ then\left(\sum_{s>u}^{t-1}\sum_{i} E_{a,i,s,v} - \sum_{i} TR_{a,v} * E_{a,i,u,v} - \sum_{i} TR_{a,v} * E_{a,i,t-1,v}\right) \\ else\left(\sum_{s>u}^{t+T-u}\sum_{i} E_{a,i,s,v} - \sum_{i} TR_{a,v} * E_{a,i,u,v} - \sum_{i} TR_{a,v} * E_{a,i,t+T-u-1,v}\right) \end{array}\right)\right)$$

We also modify the "trawler start constraint" as follows. It ensures that a trawler will go fishing or stay in the port according to the requirement and profitability of the fishery. Here if $t > u$ then the trip length is less than or equal to the maximum days in sea, i.e., $t - u \le N_v$ and if $u > t$ then the trip length $T + t - u \le N_v$.

$$for(a,1,t,v) \in trips, \sum_{a}\sum_{p}\sum_{t=2}^{N_v} w_{p,a,1,t,v} + for\ u > t\ and\ (a,u,t,v) \in trips,$$

$$\sum_{a}\sum_{p}\sum_{u=1}^{T}\sum_{t=2}^{T} w_{p,a,u,t,v} + wr_{1,v} = 1 \tag{4.7}$$

Along with these modifications, we ran the same 10-period model assuming that the trawler does not need to be at the port at the beginning and end of a planning horizon i.e., after landing its catch the trawler will continue to its next trip. A sample trawler schedule of the 10-period model is shown in Figure 4.2.

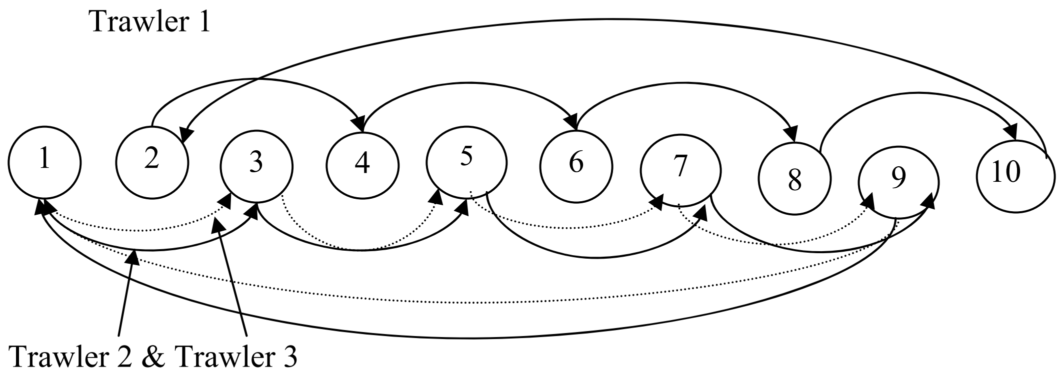

Figure 4.2: A sample fishing trawler scheduling for a 10-period model.

The 10-period model with this new assumption yields 14% more profit than that of the original problem. This is because the processing firm received 2 landings in period 1 and 1 landing in period 2. As a result the fishery is able to allocate regular labour more smoothly than under the previous assumption. The factory receives the catch from 3 more trawler trips, resulting in 22% more raw fish for processing. So the factory uses 26% more labour hours for processing these raw materials. The model now uses safety stock more smoothly than does the original model. The 10-period model uses only 2108 kilograms of initial inventory as safety stock.

Changing the length of the planning horizon, we ran different planning horizon models. A 12-period model yields 7% more profit than the original 12-period model. A 15-period model yields about 2% more profit than that of the original 15-period model.

A 30-period model yields about 1.2% more profit than that of the original 30-period model. The processing firm receives the catch from 3 more trips, resulting in a 2% more raw materials.

We also observed that the short planning horizon faces more problem than the longer planning horizon because within this short planning horizon the trawlers have to be in port at the beginning and end of each period.

We also performed the same experiments with the three other problem instances: IFPMS, IFPML, and IFPMXL. The results are consistent with those for the original problem data. For example, a 10-period IFPMS model with this new assumption yields 12% more profit than that of the original problem instance. Again, this is because the processing firm received one landing during period 1 and one landing during period 2. As a result the fishery is able to allocate regular labour more smoothly than under the previous assumption. The factory receives 2 more trawler trips and 16% more raw fish for processing, so the factory uses 21% more labour hours for processing these raw materials. The model continues to use safety stock more smoothly than under the original assumption. In Table 4.8, we summarise the results from different planning horizon models.

Problem	Characteristics			
	% profit changes	Higher trawler trips	% changes in labour	% change in raw fish
10-period	14	3	26	22
12-period	7	3	9.8	11
15-period	2	3	1.7	1.5
30-period	1.2	3	2.8	2
10-period IFPMS	12	2	21	16

Table 4.8: Summary of the results from different planning horizon models.

From all the above experiments, we observe that, unlike with the previous assumption with the repeated (cycling) trawler schedule, the fishery needs a much lower inventory of raw materials. We also observe that the longer the planning horizon, the lower the differences in the objective function values.

4.7 Generating some more data sets

Since catch rate is the most important "global" parameter which influences the profit of the fishery significantly, in this section, we will vary the mean catch rate and observe its effect on the profit. We first decrease the mean catch rate by 50% and then by 66.6%. Finally, we create some more challenging data sets assuming that some part of the planning horizon have zero catch rate to observe the impact on the effectiveness of our IFPM.

4.7.1 Mean decreased by 50%

We first decrease the mean by 50% and run the 10-period original model. Since the mean catch rate is decreased, it takes a longer time for the trawlers to reach their storage capacity. The total number of trawler trips is decreased by 1, and, as a result, the total profit of the deterministic model is reduced to $837,167, a decrease of 21.4%.

We performed the above experiments with three other different problem instances (IFPMS, IFPML and IFPMXL). The results are consistent with those of the above problem instance.

We then tried to solve a 30-period original problem with the catch rate halved, but we abandoned the solution process after more than one hour. As discussed in Section 4.2.3, the longer planning horizon model would require a lot of memory. We face the same problem with the 30-period planning horizons of IFPMS, IFPML, and IFPMXL.

For the purpose of comparison, we now apply the DBP and RCBP methods developed in Chapter 5 and Chapter 7 to solve this 30-period problem. We did not apply them for the shorter planning horizon instances, because they can be solved easily by the CPLEX. It takes 474 seconds by the DBP method and 328 seconds by the RCBP method to solve a 30-period problem instance. Table 4.9 shows the results from different planning horizon models by different methods. The profit from the RCBP method is better than that of the DBP method and required less time to obtain; it is within 0.01% of the CPLEX profit, and took less than 10% of the time to obtain.

Planning horizon	Solution methods					
	CPLEX		DBP		RCBP	
	Solution time (s)	Profit ($)	Solution time (s)	Profit ($)	Solution time (s)	Profit ($)
10-period	4	837,167	-	-	-	-
20-period	74	1,588,719	-	-	-	-
30-period	**3,600	1,907,570	474	1,857,381	328	1,905,180

Table 4.9: Solution time and profit from different planning horizons by different methods with catch rate reduced by 50%.

** The solution process was abandoned after one hour, so the solution shown is not ideally to be optimal.

In the next section, we will study the effect of variation in catch rate on profit by reducing the mean catch rate by 66.6%.

4.7.2 Mean decreased by 66.6% (Two-thirds of the mean)

In this section, we decrease the mean by 66.6% and run the 10-period original model. Since the mean catch rate is decreased too much relative to the original mean catch rate, the length of each trawler trip is increased, and so the total number of trawler trips is decreased by 4. And as a result the total profit of the deterministic model is reduced to $668,134, a decrease in the profit of 37.3%.

We then tried to solve a 30-period original problem with the one-third catch rate, but we abandoned the solution process after more than one hour. We then applied the

DBP and RCBP methods to solve this 30-period problem. It takes 340 seconds by DBP and 267 seconds by RCBP method to solve this 30-period planning horizon model. Table 4.10 shows the results from different planning horizon models by different methods. The profit from the RCBP method is again better than that from the DBP method and required less time to obtain; it is within 0.0001% of the CPLEX profit, and took less than 10% of the time to obtain.

Planning horizon	Solution methods					
	CPLEX		DBP		RCBP	
	Solution time (s)	Profit ($)	Solution time (s)	Profit ($)	Solution time (s)	Profit ($)
10-period	4	668,134	-	-	-	-
20-period	10	1,380,387	-	-	-	-
30-period	**3,600	1,639,630	340	1,556,910	267	1,639,621

Table 4.10: Solution time and profit from different planning horizons by different methods with catch rate reduced by 66%.

** The solution process was abandoned after one hour, so the solution shown is unlikely be optimal.

4.7.3 Generating some more challenging data

In the previous section, we decreased the mean catch rate by a certain percentage. In this section, we will investigate the effect of some unusual situations when there is no catch for some part of the planning horizon. For example, at the beginning or in the middle or at the end of a planning horizon, there could be a storm which can stay for several days and significantly affect profitability, or make it impossible for fishing. Investigating such situations allows us to observe the effectiveness of our deterministic model and its methods of solution in dealing with those challenging situations.

(i) Every second period with zero catch rates

We create a representative data set in which we assume that the every second period the catch rate is zero. For example, we ran a 10-period model assuming that each of the 2^{nd}, 4^{th}, 6^{th}, 8^{th} and 10^{th} periods has zero catch rate.

The total number of trawler trips decreases by 8%. The trawlers landed 21% less raw fish than that of the original problem. Since the trawlers landed fewer raw materials, the model uses 23% less labour hours per period. As a result the total profit of the deterministic model is reduced to $837,166 resulting in a decrease of 21%. We also notice longer trip lengths.

We then tried to solve a 30-period original problem assuming every second period the catch rate is zero. It took 5140 seconds to solve the model directly by the CPLEX where as the DBP method takes 336 seconds and the RCBP method takes 219 seconds to solve the same 30-period problem instance. We also solve a 20-period model with the same assumption. We present results from different planning horizon models by different methods in Table 4.11. For the 20-period instance, the DBP method obtains a superior profit to that of the RCBP method, but requires more than four times the solution time. For the 30-period instance, the RCBP method's profit is superior and found in less time. For both the problem instances, the RCBP method quickly yielded profits within 0.01% of optimality.

Planning horizon	CPLEX		DBP		RCBP	
	Solution time (s)	Profit ($)	Solution time (s)	Profit ($)	Solution time (s)	Profit ($)
10-period	4	837,166	-	-	-	-
20-period	92	1,594,269	285	1,589,386	65	1,580,386
30-period	5,140	1,903,180	336	1,889,466	219	1,900,860

Table 4.11: Solution time and profit from different planning horizons by different methods with every second period with zero catch.

(ii) Catch rates depending on the previous periods

We create another representative data set in which we assume that the catch rate depends on that of the previous period. For example, we ran a 10-period model starting with average expected catch rate in the first period. Then the catch rate starts to decrease gradually by 1/5th of the mean each time and becomes zero on the fifth period; then the catch rate gradually increases by the same constant amount every period and on the 10th period it becomes the average expected catch. The total number of trawler trips is decreased by 8%. The trawlers landed 15% less raw fish than that of the original problem. Since the trawlers landed fewer raw materials, the model uses 17% fewer labour hours per period. As a result the total profit of the deterministic model is reduced to $902,666 resulting in a decrease of 6%. We also notice longer trip lengths in the middle of the planning horizon.

We then tried to solve a 30-period original problem assuming every second period the catch rate is zero. It took 64 seconds to solve the 30-period model. We also solve a 20-period model with the same assumption. We present results from different planning horizon models in Table 4.12.

Planning horizon	Solution time (S)	Profit ($)
10-period	4	902,666
20-period	15	1,658,594
30-period	64	1,908,417

Table 4.12: Solution time and profit from different planning horizons with catch rate depending on the previous periods.

(iii) First half with mean catch and second half with zero catch

We create a representative data set in which the first half of the planning horizon has the average expected catch as mean catch and the second half of the planning horizon has zero catch. For example, we ran a 10-period model assuming that during the first five periods the trawlers can get the average expected catch as the mean catch rate and during the second five periods the weather is too bad for catching fish.

The total number of trawler trips is decreased by 25%. The trawlers landed 35% less raw fish than that of the original problem. Since the trawlers landed fewer raw materials, the model uses 38% fewer labour hours per period. As a result the total profit of the deterministic model is reduced to $716,189 resulting in a decrease of 32%. We notice that each trawler has to return to the port even they are not full because there is no catch in the final periods.

We then solve a 30-period original problem. It took 495 seconds to solve the model directly by the CPLEX whereas the DBP method takes 432 seconds and the RCBP method takes 238 seconds to solve the same 30-period problem instance. We present results from different planning horizon models by different methods in Table 4.13. The DBP method yielded a profit 0.01% superior to that of the RCBP method but took a longer time to solve. Both of the methods yielded profits within 0.01% of optimality.

Planning horizon	Solution methods					
	CPLEX		DBP		RCBP	
	Solution time (s)	Profit ($)	Solution time (s)	Profit ($)	Solution time (s)	Profit ($)
10-period	5	716,189	-	-	-	-
20-period	7	1,315,099	-	-	-	-
30-period	495	1,549,180	432	1,543,361	238	1,543,192

Table 4.13: Solution time and profit from different planning horizons by different methods with second half with zero catch.

(iv) First half with zero catch and second half with mean catch

We create another representative data set in which we assume that during the first half of the planning horizon the catch rate is zero and during the second half of the planning horizon has the average expected catch is constant i.e., the mean catch. For example, we ran a 10-period model assuming that during the first five periods the weather is too bad to go out for fishing and in the second five periods the trawlers can catch fish at the average expected catch rate.

The total number of trawler trips is decreased by 50%. The trawlers landed 50% less raw fish than in of the original problem instance. Since the trawlers landed fewer raw materials, the model uses 52% less labour hours per period. As a result the total profit of the deterministic model is reduced to $555,022 resulting in a decrease of 48%. As expected, the processing firm has to wait for seven days to receive its first landings for processing.

We also ran a 30-period model assuming that during the first fifteen periods the weather is too bad for the trawlers to leave the port and during the second fifteen periods the trawlers can catch fish at the average expected catch rate. The total number of trawler trips is decreased by 42%. The trawlers landed 40% less raw fish than that of the original problem. Since the trawlers landed fewer raw materials, the model uses 10% less labour hours per period. As a result the total profit of the deterministic model is reduced to $1,433,070 resulting in a decrease of 37%. We also notice that the processing firm has to wait for seventeen days to get the first landings. This is because there was no catch for the first fifteen periods. So the optimal solution is similar to a 15-period instance with mean catch. The 30-period model takes 25 seconds to solve directly by the CPLEX. We present results from different planning horizon models in Table 4.14.

Planning horizon	Solution time (S)	Profit ($)
10-period	4	555,022
20-period	6	1,106,015
30-period	25	1,433,070

Table 4.14: Solution time and profit from different planning horizons with first half with zero catch.

(v) First and final parts with mean catch, middle part with zero catch

We create another representative data set in which the first part and last part of the planning horizon have a constant catch rate at the mean catch, and the middle part of the planning horizon has zero catch available. For example, we ran a 10-period model assuming that in the first three periods the trawlers can catch fish at the average expected catch rate, the second four periods the weather is too bad for fishing so there is no catch for these four periods and in the last three periods the trawlers can catch fish at the average expected catch rate.

The total number of trawler trips is decreased by 25%. The trawlers landed 33% less raw fish than that of the original problem. Since the trawlers landed fewer raw materials, the model uses 35% fewer labour hours per period. As a result the total profit of the deterministic model is reduced to $741,333 resulting in a decrease of

30%. We also notice that in the middle of the planning horizon when catch rate is zero, trawlers have to return to the port with less total catch.

We then solve a 20 and a 30-period model using the assumption that the first and the last part of the planning horizon have a constant catch rate at the mean catch, and the middle part of the planning horizon has zero catch available. We present results from different planning horizon models in Table 4.15.

Planning horizon	Solution time (S)	Profit ($)
10-period	4	741,333
20-period	18	1,620,649
30-period	52	1,866,430

Table 4.15: Solution time and profit from different planning horizons with middle part with zero catch.

In another example, we ran a 10-period model assuming that in the first two periods the trawlers can catch fish at the average expected catch rate, then from 3rd to 9th periods the weather is too bad for fishing so there is no catch for these seven periods and in the last period the trawlers can catch fish at the average expected catch rate.

The total number of trawler trips is decreased by 75%. The trawlers landed 73% less raw fish than that of the original problem. Since the trawlers landed fewer raw materials, the model uses 74% less labour hours per period. As a result the total profit of the deterministic model is reduced to $293,929 resulting in a decrease of 72%. We also notice that there are no landings after 4th period.

We then solve a 30-period model assuming that the first and last five periods of the planning horizon have a constant catch rate at the mean catch, and the middle 10 periods of the planning horizon has zero catch available. It takes 10 seconds to solve this problem. We notice that there is no landing between 10 to 27th periods. As a result, the total profit decreased to $1,110,270.

From all the above experiments, we found two conclusions. First, our IFPM is able to handle those challenging situations, i.e., the IFPM is robust to these challenging data sets. Second, we found that most of the cases the longer planning horizon models are hard to solve. But the pricing methods developed in this thesis are efficient to solve these longer planning horizon problems.

4.8 Conclusion

In this chapter, we presented the structure of the IFPM and presented the computational time to show it was hard to solve. We then applied the rolling horizon approach for the solution of a 30-period planning horizon model. From the four different problem instances, we found that the classic approach of a rolling horizon was ineffective in the sense that it took long time to solve the 30-period IFPM, and reduced profits significantly as expected. The rolling horizon was intended to reduce the size of the problem to make it solvable. But it was proved to be ineffective to reduce the problem size and either took longer time to solve the longer planning horizon problems or the solutions were far from the optimum. So this is not a good way to decompose the IFPM and if the management tries to operate that way, they will be about 9% away from the optimum. We also found that the smoothened allocation of quota can reduce the solution gap but still about 5% from the optimum. Alternatively, we will develop solution techniques for solving the longer planning horizon models directly.

We also proposed a safety stock approach to deal with the end-effects-of-planning horizon. Experimenting with different challenging problem instances, we discovered that the safety stock is a simple method to manage man-made variability, due to lumpy schedules and to deal with end-effect of planning horizon.

We also investigate the effect of initial and final position of the trawlers on the profit. From the solution of different planning horizon models, we found that, unlike with the previous assumption, with the repeated (cycling) trawler schedule, the fishery needs a much lower inventory of raw materials. We also observed that, the longer the planning horizon, the lower the objective function value differences.

In terms of solving challenging problem instances of the IFPM, we found that, the performance of the RCBP method usually dominated that of the DBP method, but not always. Further, the RCBP method always took less time than the DBP method and consistently yielded profits within 0.01% of the optimality. From this preliminary testing, we can conclude that it appears the IFPM can be solved robustly.

This still leaves the problem of solving a large IFPM for a long planning horizon. We next present column generation approaches to help solve the IFPM quickly. We develop a new decomposition-based pricing (DBP) method in Chapter 5; this method was foreshadowed in the computational experiments conducted in the current chapter. In Chapter 6 and 7, we develop a decomposition-based O'Neill pricing (DBONP) method, and a reduced cost-based pricing (RCBP) method for solving the IFPM. Note that, the RCBP method was also induced in this chapter's computational experiments.

Chapter 5

Relaxation and Decomposition Methods for Solving IFPM

5.1 Introduction

In Chapter 4, we discussed that the deterministic integrated fishery planning model (IFPM) is hard to solve. It consists of a trawler scheduling subproblem, a processing subproblem, and complicating side constraints. Since the IFPM is a difficult mixed integer model, we have been led to develop solution procedures that not only work very quickly, but are very close to the optimality on the problems which we have been studying.

In this chapter, we investigate linear programming (LP) relaxation, Lagrangean relaxation (LR) with subgradient optimization (SO), Dantzig-Wolfe decomposition (DWD) and decomposition-based pricing (DBP) solution methods to gain insight into the effectiveness of these methods as a mechanism to solve the large scale IFPM. We find that the conventional decomposition techniques including subgradient optimization (SO) and DWD are unacceptably slow. We then develop a new DBP method for solving the large IFPM, which gives excellent computation times. Numerical results for several planning horizon models are presented.

The natural complexity of MILP has led to much research in approximation methods for these problems (Lubbecke & Desrosiers (2005)). Computing bounds is an essential element of the commonly used branch and bound algorithm. The bounds are generally computed by solving either a relaxed or dual bounding subproblem of the original problem. Although the bound from the LP relaxation is commonly used, it is often too weak to be effective. LR has been widely used for about two decades in many practical applications. Well-chosen LR strategies usually provide tighter bounds than those of LP relaxation. LR is based on the existence of complicating constraints that increase the complexity of the solution approach. If the MILP has two (or more) independent subproblems and some linking constraints, then one can split these independent subproblems and dualize the linking constraints.

The literature on LR and its application is enormous. Geoffrion (1974) introduced the term "Lagrangean relaxation," developed relevant theories and explored its usefulness in the context of branch-and-bound methods for mixed integer linear programs. Fisher (1981) reviewed LR and documented a number of successful applications of this method. Lubbecke and Desrosiers (2005) reviewed LR and column generation approaches for solving integer programs.

The remainder of this chapter is organized as follows. In Section 5.1.1, we briefly present the IFPM. Section 5.2 presents the LP relaxation and 5.3 presents a LR for the

IFPM. Section 5.4 describes the LR and SO. In Section 5.5, we present DWD. Section 5.6 contains a modified DWD and in Section 5.7, we present our DBP procedure.

5.1.1 IFPM in matrix-vector notation

For convenient description, we briefly re-present our IFPM in matrix notation as follows:

$$
\begin{array}{lllllll}
(P) & \text{maximize} & c^1 w & + c^2 f & + c^3 x, \text{ subject to} & & \\
 & & A^0 w & + f & & = 0. & (5.1) \\
 & & D^1 w & & & = b^1. & (5.2) \\
 & & & & D^2 x & = b^2. & (5.3) \\
 & & & A^1 f & + A^2 x & = b^0. & (5.4) \\
 & & w \in \{0,1\} & & & & (5.5a) \\
 & & & & f,\ x \geq 0. & & (5.5b)
\end{array}
$$

Equation (5.1) represents the relationship of the trawler scheduling variables w to landed fish f, as a mass balance in movement of fish from trawlers to the factory. Equation (5.2) expresses the constraints involving only trawler scheduling, indicating, for example, that a trawler may be in only one place at a time. Equation (5.3) expresses the fish processing constraints, modelling the flow of fish through the factory as raw fish is made into various products. Equation (5.4) represents the mass balance constraints, representing the flow of raw landed fish inventory into the fish processing factory. Using LR, we can relax these side constraints. It will then be easier to solve, and the objective value will be an upper bound (since it is a maximization problem) on the optimal value of the IFPM.

Through out this thesis, we will use the following notation:

$F(PR_\lambda)$ is the set of feasible solutions of the LR *(PR_λ)*.

$F(P)$ is the set of feasible solutions of the integer program *P*.

$v(PR_\lambda)$ is the value of the objective function of the LR PR_λ.

$v(P)$ is the value of the objective function of the integer program *P*.

$v(\overline{P})$ is the value of the objective function of the linear program $\overline{P}$.

$v(PR_{\bar{\lambda}})$ is the value of the objective function of the LR $PR_{\bar{\lambda}}$.

5.2 The LP relaxation

In this section, we will discuss the linear programming (LP) relaxation for the IFPM. When the integer constraints (5.5a) are relaxed from *P*, the model is designated as $\overline{P}$**,** the usual LP. To compare the bounds of the IFPM with LP relaxation $\overline{P}$ and *P* itself, we solve a 10-period model of the original problem and notice that the integer optimal value is *v(P)* = $ 1,065,775 and the optimal value of the linear programming relaxation is $v(\overline{P})$ = $1,066,350. That is $v(\overline{P}) \geq v(P)$ with duality gap 0.05%.

The LP relaxation of the 10-period model of the original problem yields a fractional solution for the binary trawler scheduling variables $w_{p,a,u,t,v}$, presented on Table 5.1.

w[timaru, area3, 1, 3, tr1] = 0.928365 w[timaru, area3, 1, 4, tr1] = 0.0716354 w[timaru, area3, 3, 5, tr1] = 0.526813 w[timaru, area3, 3, 6, tr1] = 0.401552 w[timaru, area3, 4, 6, tr1] = 0.0716354 w[timaru, area3, 5, 7, tr1] = 0.0629693 w[timaru, area3, 5,8,tr1] = 0.463844 w[timaru,area3, 6, 8, tr1] = 0.473187 w[timaru,area3,7, 10, tr1] = 0.0629693 w[timaru, area3, 8, 10, tr1] = 0.937031	w[timaru, area3, 2, 4, tr2] = 1 w[timaru, area3, 4, 6, tr2] = 1 w[timaru, area3, 6, 8, tr2] = 1 w[timaru, area3, 8, 10, tr2] = 1 w[timaru, area3, 2, 4, tr3] = 1 w[timaru, area3, 4, 6, tr3] = 1 w[timaru, area3, 6, 8, tr3] = 1 w[timaru, area3, 8, 10, tr3] = 1.

Table 5.1: LP relaxation solution of scheduling variables

Though the duality gap is very small with a 10-period problem, it is not feasible for the IFPM as the binary variables *w* have fractional values. The reason for this is of course “why pay full cost if you are not using the whole day for fishing?” Table 5.2 compares the IP solutions to the LP relaxation solutions of the original problem showing the percentage duality gap. The duality gap for some of the planning horizon is high. For example, a 13, 17, 23-period model has higher duality gap each above 2%. For millions of dollars it is a big duality gap.

Planning Horizon	$v(P)$ (\$)	$v\left(\bar{P}\right)$ (\$)	% Duality gap
5	522,764	522,764	0.00
6	556,945	557,440	0.09
7	770,767	770,767	0.00
8	812,587	813,076	0.06
9	1,013,345	1,013,345	0.00
10	1,065,775	1,066,350	0.05
11	1,255,777	1,255,777	0.00
12	1,313,945	1,314,621	0.05
13	1,431,831	1,466,321	2.35
14	1,506,253	1,515,077	0.58
15	1,582,008	1,607,944	1.61
16	1,621,743	1,648,103	1.60
17	1,695,835	1,734,379	2.22
18	1,746,724	1,774,867	1.59
19	1,826,217	1,859,060	1.77
20	1,880,196	1,898,411	0.96
21	1,931,858	1,963,397	1.61
22	1,962,473	1,992,527	1.51
23	2,007,252	2,056,248	2.38
24	2,048,128	2,084,239	1.73
25	2,121,887	2,141,757	0.93
26	2,146,273	2,173,053	1.23
27	2,192,681	2,220,159	1.24
28	2,236,589	2,258,272	0.96
29	2,261,176	2,295,345	1.49
30	2,300,871	2,331,036	1.29

Table 5.2: Comparison of IP solutions to the LP relaxation solutions of the original problem.

Problem	PH	IP profit ($)	LP profit ($)	% Duality gap
IFPMS	5	335,477	335,477	0
	10	701,182	702,866	0.01
	15	1,123,295	1,123,295	0
	20	1,439,023	1,461,530	1.5
	25	1,705,280	1,733,364	1.6
	30	1,864,290	1,905,126	2.1
IFPML	5	660,701	665,741	0.07
	10	1,347,194	1,353,447	0.04
	15	1,852,260	1,891,241	2.06
	20	2,196,291	2,234,440	1.7
	25	2,400,920	2,444,345	1.8
	30	2,550,260	2,605,895	2.1
IFPMXL	5	732,706	747,420	1.9
	10	1,542,810	1,554,154	0.7
	15	1,994,834	2,006,230	0.6
	20	2,248,057	2,262,451	0.6
	25	2,396,554	2,416,450	0.8
	30	2,546,817	2,568,376	0.8

Table 5.3: Comparison of IP solutions to the LP relaxation solutions of IFPMS, IFPML and IFPMXL.

Table 5.3 compares the IP solutions to the LP relaxation solutions of the three different problems IFPMS, IFPML, and IFPMXL showing the percentage duality gap. From Table 5.1 to 5.3, we found that the LP relaxations are too weak to be effective in the sense that, this is not feasible for the IFPM as the binary variables *w* have fractional values. The reason for this is of course “why pay full cost if you are not using the whole day for fishing?” Also, in real life it is not possible to complete a trawler’s trip for fishing within a quarter of a day. In the following section, we present Lagrangean relaxation (LR) for solving the IFPM.

5.3 LR for the IFPM

To obtain the LR of *P*, we associate the Lagrangean multipliers θ with the complicating constraints of the IFPM, and bring this term into the objective function as follows:

(PR_θ)

$$\operatorname*{Max}_{f,w,x}\left\{\begin{array}{l} c^1 w + c^2 f + c^3 x - \theta\left(A^1 f + A^2 x - b^0\right) \mid A^0 w + f = 0, D^1 w \le b^1, D^2 x \le b^2, \\ w, f \in S^1, x \in S^2 \end{array}\right\}.$$

We now present the LR of the IFPM in algebraic notation in detail. For all *i*, *l*, and *t*, let $\theta_{i,l,t}$ be the Lagrangean multipliers for the inventory balance constraint set (3.5). Then the LR of the IFPM can be defined as:

(PR_θ) Maximize

$$-\left(\sum_p\sum_a\sum_t\sum_u\sum_v (t-u) V_{t,v} w_{p,a,u,t,v} + \sum_i\sum_l\sum_t\sum_v\sum_a C_{a,i,t,v} f_{a,i,l,t,v} + \sum_i\sum_l\sum_t I_t z_{i,l,t}\right)$$
$$+\left(\sum_i\sum_j\sum_l\sum_t P_{i,j,l} s_{i,j,l,t} - \sum_t Lr_t\, yr - \sum_t Lo\, yo_t - \sum_i\sum_j\sum_l\sum_t J_t r_{i,j,l,t}\right)$$
$$-\sum_{i,l,t}\theta_{i,l,t}\left(z_{i,l,t-1} - z_{i,l,t} + \sum_a\sum_v f_{a,i,l,t,v} - \sum_j F_{i,j} x_{i,j,l,t}\right)$$

subject to 3.1 – 3.4, 3.6 – 3.9, 3.13, 3.14a, 3.14b, and 4.1 – 4.3

The Lagrangean relaxation PR_θ decomposes into independent problems. The first one is a trawler scheduling problem involving trawler scheduling and quotas and is denoted by $PR1_\theta$. Some variables in the trawler scheduling constraints are binary. The other problem is a production problem involving processing and labour allocations and is denoted by $PR2_\theta$. If at least one of these subproblems does not hold the "Integrality Property" (Geoffrion, 1974, Fisher, 1981), i.e. if the optimal value of $PR1_\theta$ or $PR2_\theta$ is altered by dropping the integrality condition, then the LR may yield a tighter bound than the LP bound.

($PR1_\theta$) Maximize

$$-\left(\sum_p\sum_a\sum_u\sum_t\sum_v (t-u)V_{t,v}w_{p,a,u,t,v} + \sum_i\sum_l\sum_t\sum_v\sum_a C_{a,i,t,v}\, f_{a,i,l,t,v} + \sum_i\sum_l\sum_t I_t z_{i,l,t}\right)$$
$$-\sum_{i,l,t,k}\theta^k_{i,l,t}\left(z_{i,l,t-1} - z_{i,l,t} + \sum_a\sum_v f_{a,i,l,t,v}\right)$$

subject to 3.1 – 3.4, 3.6, and 3.13.

$f_{a,i,l,t,v}$, $z_{i,l,t}$, $w_{p,a,u,t,v}$, $wr_{t,v}$, $q_{a,i,t} \geq 0$ and

$w_{p,a,u,t,v}$, $wr_{t,v} \in \{0,1\}$.

In matrix-vector notation it can be written as

$$PR1_\theta: \operatorname*{Max}_{f,w,x}\left\{c^1 w + c^2 f - \theta\left(A^2 x - b^0\right) \mid A^0 w + f = 0, D^1 w \leq b^1\right\}.$$

and ($PR2_\theta$) Maximize

$$\left(\sum_i\sum_j\sum_l\sum_t P_{i,j,l}\, s_{i,j,l,t} - \sum_t Lr_t\, yr - \sum_t Lo\, yo_t - \sum_i\sum_j\sum_l\sum_t J_t r_{i,j,l,t}\right)$$
$$-\sum_{i,l,t,k}\theta^k_{i,l,t}\left(\sum_j F_{i,j}\; x_{i,j,l,t}\right)$$

subject to 3.7 – 3.9, and 4.1 – 4.3.

$x_{i,j,l,t}$, $s_{i,j,l,t}$, $r_{i,j,l,t}$, yr, yo_t, ≥ 0.

In matrix-vector notation it can be written as

$$PR2_{\theta}: \underset{f,w,x}{\text{Max}}\left\{c^{3}x-\theta\left(A^{1}f-b^{0}\right) \mid D^{2}x \leq b^{2}\right\}.$$

Problem $PR1_{\theta}$ is an integer program and problem $PR2_{\theta}$ is a linear program. From the solution of different planning horizon models with $PR1_{\theta}$, we observed that it does not hold the "Integrality Property," since the LP relaxation of these problems does not always have an integer optimal solution (Desrosiers et. al., 1988). For example, Table 5.4 shows that the solution of a 15-period $PR1_{\theta}$ has 14.3% non-integer values. So $PR1_{\theta}$ is not naturally integer. The LR scheme thus yields a stronger bound than the LP relaxation bound.

w[timaru, area7, 1, 5, tr1] = 1 w[timaru, area7, 5, 9, tr1] = 0.367128 w[timaru, area3, 5, 7, tr1] = 0.632872 w[timaru, area3, 7, 9, tr1] = 0.632872 w[timaru, area3, 9, 11, tr1] = 1 w[timaru, area3, 11, 13, tr1] = 1 w[timaru, area3, 13,15,tr1] = 1 w[timaru,area3, 1, 3, tr2] = 1 w[timaru,area3,3, 5, tr2] = 1 w[timaru, area3, 5, 7, tr2] = 1 w[timaru, area3, 7, 9, tr2] = 1	w[timaru, area3, 9, 11, tr2] = 1 w[timaru, area3, 11, 13, tr2] = 1 w[timaru, area3, 13, 15, tr2] = 1 w[timaru, area3, 1, 3, tr3] = 1 w[timaru, area3, 3, 5, tr3] = 1 w[timaru, area3, 5, 7, tr3] = 1 w[timaru, area3, 7, 9, tr3] = 1. w[timaru, area3, 9, 11, tr3] = 1 w[timaru, area3, 11, 13, tr3] = 1 w[timaru, area3, 13, 15, tr3] = 1

Table 5.4: An LP relaxation solution of $PR1_{\theta}$ for the $w_{p,a,u,t,v}$ variable.

How is θ to be chosen?

The potential usefulness of LR is largely determined by how near its optimal solution is to that of the integer program *P*. This necessitates a criterion for choosing an appropriate θ. The ideal choice would be to take θ as an optimal solution of the Lagrangean dual *D* (Fisher 1981). In the following section, we discuss LR and SO to solve the IFPM.

5.4 LR and subgradient optimization (SO)

The commonly used method of finding the optimal multipliers in LR is SO (Held et. al., 1974, Shepardson & Marsten (1980), Wolsey (1998)). This approach yields the Lagrangean multiplier θ directly. This method has proven effective in practice for a variety of problems. It is possible to choose the step size t^k to guarantee convergence to the optimal solution. Approximating the LP bound on the original problem by LR, can be advantageous for several reasons. Most important is the speedup of computation by solving easier subproblems. Another reason may be the computation of feasible solutions. During SO, one gets a series of solutions which may violate the relaxed constraints. Since violated constraints tend to have larger Lagrangean multipliers associated with them during SO, they are more likely to become satisfied in later iterations of the algorithm. This often results in a good feasible solution for the original problem. But despite all this optimism, the SO took longer time to solve longer planning horizon IFPM and the solution was far from the optimal. Because this method in not finite and it is very difficult to establish a suitable stopping criterion (Desrosiers et. al., 1988).

In Section 5.4.1, we present an algorithm for SO to solve the IFPM, by relaxing the complicating inventory balance constraint (3.5). We choose to relax this inventory balance (3.5) constraint because it contains variables from both the subproblems, and was therefore the complicating constraint. In Section 5.4.2, we solve the same problem by relaxing the landed fish constraints (3.1). We choose to relax the landed fish (3.1) constraint because relaxing this constraint allows separation of the networking constraints from the trawler scheduling subproblem.

5.4.1 Relaxation of inventory balance constraint

The subgradient optimization algorithm for the IFPM can be summarised as follows where $\theta^k_{i,l,t}$ is Lagrangean multiplier.

Step1. Initialize iteration $k = 0$ and set jump size t^k, $slack = 0$.

Choose an initial $\theta^k_{i,l,t}$ as the dual value of the complicating constraint obtained by solving $\overline{P}$.

Step2. Solve PR_θ for $\theta^k_{i,l,t}$.

Step3. For *i, l, t*, let $\theta^{k+1}_{i,l,t} = \theta^k_{i,l,t} + t^k * (z_{i,l,t-1} + \sum_a \sum_v f_{a,i,l,t,v} - \sum_j F_{i,j}\, x_{i,j,l,t} - z_{i,l,t})$.

Step4. Set $t^{k+1} = t^k * 0.9998 *$ (Lagrangean value – LP value)/ slack, where

$$slack = slack + (z_{i,l,t-1} + \sum_a \sum_v f_{a,i,l,t,v} - \sum_j F_{i,j}\, x_{i,j,l,t} - z_{i,l,t})^2.$$

Step5. If | $\theta^{k+1}_{i,l,t}$ - $\theta^k_{i,l,t}$ |< ε then stop. Else if maximum number of iterations was reached then stop. Else $k = k + 1$ and go back to step 1.

In Chapter 3, we solved the 10-period IFPM as an IP (with the integrality restrictions (3.14b) included) which yielded an optimal value of $1,065,775. Relaxing the inventory balance constraint (3.5), we solve the same 10-period problem using an initial $\theta^0_{i,l,t}$ obtained by solving an LP relaxation of the model. The model yields an optimal solution of $1,065,991. A 30-period problem using an initial $\theta^0_{i,l,t}$ obtained by solving an LP relaxation of a 30-period model yields an optimal solution of

$2,325,650 which is a better bound than that of LP relaxation. The solutions of different planning horizon models are shown in Table 5.5.

	5-period	**10-period**	**30-period**
IP optimum	$522,764	$1,065,775	$2,300,871
LP optimum	$522,764	$1,066,350	$2,331,036
SO optimum	$522,764	$1,065,991	$2,325,650
SO solution time, seconds	718	1120	3625

Table 5.5: Numerical results for SO, relaxing constraint set 5.4 of the original problem.

We then try to solve three other different problems (IFPMS, IFPML, and IFPMXL). The computational time and the profit obtained from these problems are shown in Table 5.5a.

Problem	True optimum (IP) ($)	LR profit ($)	LP profit ($)	Solution time (seconds)
IFPMS	1,864,290	1,903,258	1,905,126	3,827
IFPML	2,550,260	2,604,674	2,605,895	4,678
IFPMXL	2,540,817	2,562,495	2,568,376	5,025

Table 5.5a: Numerical results for SO, relaxing constraint set 5.4 of a 30-period IFPMS, IFPML, and IFPMXL.

Solving different planning horizon models of different problems, we found that, it is hard to find an appropriate stopping criterion. Relaxed the complicating inventory balance constraint (3.5), we tried to solve a 30-period IFPMS problem. This method took a considerably longer time (3827 sec.) and many iterations to solve PR_{θ}. In the following section, we will try this method by relaxing the landed fish constraint (3.1). We relax this constraint to allow separation of the networking constraints form the trawler scheduling subproblem.

5.4.2. Relaxation of landed fish constraint

In this section, we relax the landed fish constraint (3.1) and apply the SO algorithm to find a convenient solution procedure for the IFPM. By relaxing this constraint we separated the networking constraints from the trawler scheduling subproblem. For each relaxed constraint (3.1), we introduce a Lagrangean multiplier $\theta^k_{a,i,l,t,v}$.

$(PR3_\theta)$ Maximize

$$-\left(\sum_p\sum_a\sum_u\sum_t\sum_v (t-u)V_{t,v}w_{p,a,u,t,v} + \sum_i\sum_l\sum_t\sum_v\sum_a C_{a,i,t,v}\, f_{a,i,l,t,v} + \sum_i\sum_l\sum_t I_t z_{i,l,t}\right)$$
$$+\left(\sum_i\sum_j\sum_l\sum_t P_{i,j,l}\, s_{i,j,l,t} - \sum_t Lr_t\, yr - \sum_t Lo\, yo_t - \sum_i\sum_j\sum_l\sum_t J_t r_{i,j,l,t}\right)$$
$$-\sum_{a,i,l,t,v}\theta_{a,i,l,t,v}\left(f_{a,i,l,t,v} - + \sum_p\sum_u ET_{a,i,u,t,v}\times FR_{i,l}\times w_{p,a,u,t,v}\times FRaw_{a,i,v}\right)$$

subject to 3.2 – 3.9, 3.13, 3.14a – 3.14b, and 4.1 – 4.3.

In matrix-vector notation it can be written as

$$PR3_\theta:\ \underset{f,w,x}{\text{Max}}\left\{c^1 w + c^2 f + c^3 x - \theta(A^0 w + f) \mid D^1 w \le b^1,\ D^2 x \le b^2,\ A^1 f + A^2 x = b^0\right\}.$$

	5 periods	10 periods
IP optimum	\$522,764	\$1,065,775
LP optimum	\$522,764	\$1,066,350
LR optimum	\$522,764	\$1,070,450
SO solution time, seconds	952	1360

Table 5.6. Comparison LP and LR relaxations solutions with true optimum (IP).

It took 952 seconds to solve a 5-period model and 1360 seconds to solve a 10-period model. Results are shown in Table 5.6. We also try to solve a 30-period planning horizon model of the problems IFPMS, IFPML, and IFPMXL. It took considerably long time and many iterations to solve the 30-priod models and the results were far from the optimum.

So, from all of the above experiments, we found that the SO method is time consuming. SO has been reported to result in unpredictable convergence behaviour (Guignard, 1987) and this was the case with this model. To improve the performance of LR by SO, we experimented with modifications for updating the Lagrangean multipliers. We varied the step size and price updating formula as can be seen in the algorithm. It slightly decreases the computational time but is still not effective. So to find an efficient solution procedure we will try DWD in the following section. DWD is the next choice because it is one of the most common methods to decompose large problems (Ho & Loute, 1981, 1983).

5.5 Dantzig-Wolfe decomposition

In this section, we apply DWD (Dantzig, 1963), which is a suboptimization of the LP relaxation problem. DWD uses the convex hull portion of the constraints of the LP problem represented in terms of extreme points. According to this decomposition, the fishery generates a set of simplex multipliers for commonly-used resources. These simplex multipliers are then passed to the subproblems, which use these multipliers to generate proposed operating plans. Once all proposals have been passed to the master, it determines the best mix of proposals, and determines new simplex multipliers for the common resources. The procedure terminates when no new proposals come from the subproblems.

This decomposition may be interpreted in the following way: the fishery manager uses a master model to generate prices for raw fish. These prices are passed to the fishing trawler captains who propose trawler schedules, and to the factory manager who proposes a production schedule. Their proposals are passed to the fishery manager, who uses the master model to find the best mix of proposals and new prices for raw fish. Some resources may be used by both the sectors. These are called common resources. The master generates a set of prices (the simplex multipliers) for the commonly used resources. These prices can be passed to the factory manager and trawler captain who can use them to generate operating plans. The procedure terminates when no new proposals come from the trawler captain and factory manager.

We use the LR to relax the inventory balance constraint (3.5). Let the set of variables in the trawler scheduling subproblem be defined as $x^1 = \{w_{p,a,u,t,v}, wr_{t,v}, z_{i,l,t}, f_{a,i,l,t,v}, q_{a,i,t}\}$. Then express these variables as a convex combination of the extreme points of the feasible region for the trawler scheduling constraint set (5.6). Since all the variables of the trawler scheduling constraints are bounded, the convex set $S^1 = \{x^1 \mid D^1 x^1 = b^1, \quad x^1 \geq 0\}$ is bounded.

We can express any point x^1 in S^1 as a convex combination of the extreme points of the feasible region for S^1. If we let $x^{11}, x^{12}, \ldots, x^{1K}$ be the extreme points of this feasible region, then any point x^1 in S^1 can be expressed as a linear combination of the extreme points as follows:

$$x^1 = \sum_{k=1}^{K} \lambda^{1k} x^{1k} \tag{5.6}$$

with $\sum_{k=1}^{K} \lambda^{1k} = 1, \quad \lambda^{1k} \geq 0.$

Similarly, let the set of variables in the processing subproblem be defined as $x^2 =$ $\{x_{i,j,l,t}, s_{i,j,l,t}, r_{i,j,l,t}, yr, yo_{t,c}\}$. Since all the variables of the trawler scheduling constraints are bounded, the convex set $S^2 = \{x^2 \mid D^2 x^2 = b^2, \quad x^2 \geq 0\}$ is bounded.

$$x^2 = \sum_{k=1}^{K} \lambda^{2k} x^{2k} \tag{5.7}$$

with $\displaystyle\sum_{k=1}^{K} \lambda^{2k} = 1, \quad \lambda^{2k} \geq 0.$

By using equations (5.6) and (5.7), we represent the objective function and inventory balance constraint set (3.5) in terms of λ^{1k} and λ^{2k}. After adding the convexity constraints, and their nonnegativity restrictions, we generate the **master problem** as follows:

(M^k) Maximize

$$= -\sum_{k=1}^{K1} \lambda^{1k} \left(\sum_p \sum_a \sum_u \sum_t \sum_v (t-u) V_{t,v} w^k_{p,a,u,t,v} + \sum_i \sum_l \sum_t \sum_v \sum_a C_{a,i,t,v} f^k_{a,i,l,t,v} + \sum_i \sum_l \sum_t I_t z^k_{i,l,t} \right)$$

$$+ \sum_{k=1}^{K2} \lambda^{2k} \left(\sum_i \sum_j \sum_l \sum_t P_{i,j,l}\, s^k_{i,j,l,t} - \sum_t Lr_t\, yr^k - \sum_t Lo\, yo^k_t - \sum_i \sum_j \sum_l \sum_t J_t r^k_{i,j,l,t} \right),$$

subject to $\displaystyle\sum_{k=1}^{K1} \lambda^{1k} \left(z^k_{i,l,t-1} - z^k_{i,l,t} + \sum_a \sum_v f^k_{a,i,l,t,v} \right) - \sum_{k=1}^{K2} \lambda^{2k} \sum_j F_{i,j}\, x^k_{i,j,l,t} + Slack_{i,l,t} = 0$

for all i,l, and t, where $Slack_{i,l,t}$ is an artificial variable.

$$\sum_{k=1}^{K1} \lambda^{1k} = 1$$

$$\sum_{k=1}^{K2} \lambda^{2k} = 1$$

$$\lambda^{1k}, \lambda^{2k} \geq 0.$$

Note that λ^{1k} is continuous, so this model will only provide an upper bound.

Let $\theta_{i,l,t}$ be the dual prices associated with the inventory balance constraints (3.5).

Evaluating the columns of λ^{1k} and λ^{2k}, we obtain the reduced costs.

In matrix-vector notation, the master can be written as,

(M^k) Maximize $\sum_{k=1}^{K1} \lambda^{1k}(c^1 w + c^2 f) + \lambda^{2k} \sum_{k=1}^{K2} c^3 x$, subject to

Inventory balance rows, $$\sum_{k=1}^{K1} \lambda^{1k} \left(A^1 f\right) - \sum_{k=1}^{K2} \lambda^{2k} \left(A^2 x\right) = 0, \tag{5.8}$$

Trawler scheduling, $$\sum_{k=1}^{K1} \lambda^{1k} = 1, \tag{5.9}$$

Fish processing, $$\sum_{k=1}^{K2} \lambda^{2k} = 1, \tag{5.10}$$

$$\lambda^{1k}, \lambda^{2k} \geq 0.$$

Note that λ^{1k} is continuous, so this model will only provide an upper bound.

Let θ be the dual prices associated with the inventory balance constraint (5.8). The subproblems are as follows.

1. Trawler scheduling subproblem S_1^k, max $c^1 w + c^2 f - \theta(A^1 f)$, subject to

constraint sets (5.1) and (5.2),

$f \geq 0$ and $w \in \{0,1\}$.

2. Processing subproblem S_2^k, maximize $c^3 x - \theta(A^2 x)$, subject to

constraint set (5.3),

$x \geq 0$.

Since the trawler scheduling subproblem is not naturally integer, i.e. the optimal value of S_1^k is altered by dropping the integrality condition, the DWD upper bound should be tighter than that for LP. In the following section, we present the DWD algorithm for solving the IFPM.

5.5.1 DWD algorithm for the IFPM

The DWD method for the IFPM can be summarised as follows:

Step 0: Initialize. Set iteration k = 1 and set an initial feasible solution

$(w, wr, f, z, q, x, s, r, yr, yo)^0$.

Step 1: Solve the master M^k to obtain solutions $\lambda 1^1$ and $\lambda 2^1$, and dual price θ_1.

Step 2: Solve subproblem 1, $PR1_\theta$ (S_1^k) the trawler scheduling subproblem. Obtain

$(w, wr, f, z, q)^1$ which become coefficients of a new variable $\lambda 1^2$. Solve subproblem 2, $PR2_\theta$ (S_2^k) the processing subproblem. Obtain $(x, s, r, yr, yo)^1$ which become coefficients of a new variable $\lambda 2^2$. Calculate total profit by adding subproblems profit as; $v(P) = v(S_1^k) + v(S_2^k)$.

Step 3: If master profit equals subproblem profit, i.e., if $v(M^k) = v(P)$ then STOP.

Else go back to step 1.

The optimal values of λ^{1k} and λ^{2k} found in (5.6) and (5.7) will yield the optimal values of $x_1^1, x_2^1 ..., x_{k1}^1, x_{k1+1}^1, ..., x_k^1$ if it converges.

5.5.2 Numerical results

In Chapter 4, we solved a five period IFPM, which yielded a total profit of $522,764 for the fishery. For this section, we developed an AMPL model to run DWD for our IFPM. We first ran a model using zero initial solution and terminated after a few iterations; the solution obtained was used as an initial feasible solution. We then ran the same 5-period model starting with this initial feasible solution. It yielded the same profit as obtained earlier. In a computer with an Intel Pentium III processor with a clock speed of 665 MHz and 384 MB of RAM, it took 1,168 iterations and four hours fifty four minutes. We also ran the same model with zero feasible solution. The results are shown in Table 5.7.

	With an initial feasible solution	With zero feasible solution
Number of Iterations	1168	1068
Computation time	4:54:13	3:58:49
Subproblem1 profit	$432,160.0	$432,138.0
Subproblem2 profit	$90,603.8	$90,625.5
Total profit	$522,763.5	$522,763.5
Master profit	$522,763.5	$522,763,5

Table 5.7: Optimal values, iterations, computation times of a 5-period model solved by DWD

In either case, we observe that a small 5-period problem requires 4 to 5 hours solution time. But direct solution with CPLEX takes only 3 seconds to solve a five period model directly. So compare to the direct solution with CPLEX, we find the DWD as very slow. We further attempted to use DWD to solve models with longer planning horizons, but these took a very long time to solve.

We next try to solve the IFPM by relaxing the landed fish constraint (3.1) in the following section.

5.5.3 DWD for the relaxation of the landed fish constraint

In this section, following a similar procedure as in section 5.5.1, we try to solve the IFPM by relaxing the landed fish constraint (3.1). Let $\theta_{a,i,l,t,v}$ be the dual prices associated with the landed fish constraint (3.1), and let λ^1 and λ^2 be associated with the subproblem 1 constraints (3.2 to 3.3) and subproblem 2 constraints (3.4 – 3.9, 3.13, 4.1 – 4.3) respectively.

Subproblem 1:

$(PR4_\theta)$ Maximize

$$-\sum_p\sum_a\sum_u\sum_t\sum_v (t-u)V_{t,v}w_{p,a,u,t,v} - \sum_{a,i,l,t,v}\theta_{a,i,l,t,v}\left(\sum_p\sum_u ET_{a,i,u,t,v}\times FR_{i,l}\times w_{p,a,u,t,v}\times FRaw_{a,i,v}\right)$$

subject to 3.2 & 3.3.

$w_{p,a,u,t,v}$, $wr_{t,v} \geq 0$ and $w_{p,a,u,t,v}$, $wr_{t,v} \in \{0,1\}$.

Subproblem 2:

$(PR5_\theta)$ Maximize

$$-\left(\sum_i\sum_l\sum_t\sum_v\sum_a C_{a,i,t,v}\, f_{a,i,l,t,v} + \sum_i\sum_l\sum_t I_t z_{i,l,t}\right) +$$
$$\left(\sum_i\sum_j\sum_l\sum_t P_{i,j,l}\, s_{i,j,l,t} - \sum_t Lr_t\, yr - \sum_t Lo\, yo_t - \sum_i\sum_j\sum_l\sum_t J_t r_{i,j,l,t}\right) - \sum_{a,i,l,t,v}\theta_{a,i,l,t,v}\left(f_{a,i,l,t,v}\right)$$

subject to 3.4 – 3.9, 3.13, 4.1 – 4.3.

$f_{a,i,l,t,v}$, $z_{i,l,t}$, $q_{a,i,t}$, $x_{i,j,l,t}$, $s_{i,j,l,t}$, $r_{i,j,l,t}$, yr, yo_t, ≥ 0 .

After adding the convexity constraints, and their nonnegativity restrictions, we generate the **master problem** as follows:

Maximize

$$
= -\sum_{k=1}^{K1} \lambda^{1k} \left(\sum_p \sum_a \sum_u \sum_t \sum_v (t-u) V_{t,v} w_{p,a,u,t,v}^k \right)
$$

$$
-\sum_{k=1}^{K2} \lambda^{2k} \left(\begin{array}{l} \sum_i \sum_l \sum_t \sum_v \sum_a C_{a,i,t,v} f_{a,i,l,t,v}^k + \sum_i \sum_l \sum_t I_t z_{i,l,t}^k \\ -\sum_i \sum_j \sum_l \sum_t P_{i,j,l} s_{i,j,l,t}^k + \sum_t Lr_t \, yr^k + \sum_t Lo \, yo_t^k + \sum_i \sum_j \sum_l \sum_t J_t r_{i,j,l,t}^k \end{array} \right)
$$

Subject to

$$
\sum_{k=1}^{K1} \lambda^{2k} f_{a,i,l,t,v}^k - \sum_{k=1}^{K2} \lambda^{1k} \left(\sum_p \sum_u ET_{a,u,t,v} w_{p,a,u,t,v} FR_{i,l} FRaw_{a,i,v} \right) + Slack1_{a,i,l,t,v} = 0 \text{ for all a,}
$$

i, *l, t,* and *v* where $Slack1_{a,i,l,t,v}$ is an artificial variable.

$$
\sum_{k=1}^{K1} \lambda^{1k} = 1
$$

$$
\sum_{k=1}^{K2} \lambda^{2k} = 1
$$

$$
\lambda^{1k}, \lambda^{2k} \geq 0 .
$$

Now let $\theta_{a,i,l,t,v}$ be the dual prices associated with the landed fish constraints (3.1). Evaluating the columns of λ^{1k} and λ^{2k}, we obtain the reduced costs.

We first set an initial feasible solution $(w, wr, f, z, q, x, s, r, yr, yo)^0$ and solve the master to obtain solutions $\lambda 1^1$ and $\lambda 2^1$ for dual price $\theta_{a,i,l,t,v}^1$. Then we solve the subproblem 1 $(PR4_\theta)$ to obtain $(w, wr)^1$ which generate variable $\lambda 1^2$ and solve subproblem 2 $\left(PR5_{\theta}\right)$ to obtain $\left(f, z, q, x, s, r, yr, yo\right)^{1}$ which generate variable $\lambda 2^{2}$. If master profit equals the subproblem profit then stop. Else repeat the same process.

Using no "stock" and "factory" sets, but 3 fish species and two trawlers we solved a 3-period model. To solve the same 3-period model by DWD method with zero initial solution and by relaxing the landed fish constraint (3.1), it took 1787 iterations and yielded the same optimal objective value as we obtained from the original IP problem. We then solved the same 3-period by DWD method by relaxing the landed fish constraint (3.1) with a naively created initial solution. It took 1,367 iterations to solve the problem and yielded the same optimal objective value as we obtained from the original IP problem. Using one stock and one factory, 3 fish species, and two trawlers we solved a 3-period model as IP. The DWD method with zero initial solution took 1,787 iterations to solve the model. The same 3-period model by DWD method with an initial solution which we created naively took 1,367 iterations to solve the problem and yielded the same optimal objective value as we obtained from the original problem.

We also solve the above example with an increased length of the planning horizon of 5-periods by DWD method with the relaxation of landed fish constraint (3.1). It yielded $80 higher than that obtained from the original problem, in 3923 iterations. Finally, we tried to solve a 10-period planning horizon model by DWD method with the relaxation of landed fish constraint (3.1). After 4536 iterations, and seven hours the problem was discarded.

Severely curtailing the problem size, we observed the effectiveness of the DWD method. This proved even worse computationally. Even relaxation of the landed fish constraint (3.1) made no difference. Since the basic DWD performed badly, we conclude that DWD is not effective for the IFPM. We tried to solve the three other different problems (IFPMS, IFPML, and IFPMXL) by relaxing the complex inventory balanced constraints (3.5) and landed fish constraints (3.1). But we faced the same problems. This is because our IFPM has one integer subproblem and one linear subproblem. DWD master has two variables λ^{1k} and λ^{2k} respectively associated with each iteration of these subproblems. Since DWD master produces convex combinations of the continuous variables of the subproblems solutions, we allow λ^{1k} being continuous. So DWD provides only the upper bound for the IFPM. Also this approach adds only two variables at each iteration. So to solve a problem with thousands of variable, it takes an extremely long time. Hence we conclude that the DWD will not be effective for solving our IFPM which has thousands of variables and equations.

We then study a modification of the DWD for solving IFPM in the following section.

5.6 Modified DWD algorithm for the IFPM

In the previous section, we implemented the DWD algorithm for the IFPM using two subproblems and a master problem. In this section, we modified the DWD algorithm for the IFPM using only the trawler scheduling subproblem $(PR1_\theta)$, and using the primal processing variables and constraints directly in the master.

We express variables in the trawler scheduling subproblem $(PR1_\theta)$ x^1 as a convex combination of the extreme points of the feasible region for the trawler scheduling constraints (3.1 - 3.4, 3.6, 3.13). After adding the convexity constraints and their nonnegativity restrictions, we formulate the **master problem** as follows:

Master problem (M^k):

Maximize

$$-\sum_{k=1}^{K}\lambda^k\left(\sum_p\sum_a\sum_u\sum_t\sum_v(t-u)V_{t,v}w^k_{p,a,u,t,v}+\sum_i\sum_l\sum_t\sum_v\sum_a C_{a,i,t,v}\,f^k_{a,i,l,t,v}+\sum_i\sum_l\sum_t I_t z^k_{i,l,t}\right)$$
$$+\left(\sum_i\sum_j\sum_l\sum_t P_{i,j,l}\,s_{i,j,l,t}-\sum_t Lr_t\,yr-\sum_t Lo\,yo_t-\sum_i\sum_j\sum_l\sum_t J_t r_{i,j,l,t}\right).$$

subject to

$$\sum_{n=1}^{K}\lambda^k\left(z^k_{i,l,t-1}-z^k_{i,l,t}+\sum_a\sum_v f^k_{a,i,l,t,v}\right)-\sum_j F_{i,j}\,x_{i,j,l,t}=0 \quad \text{for all } i,\ l, \text{ and } t.$$

3.7 – 3.9, 3.13, 4.1 – 4.3.

$$\sum_{k=1}^{K}\lambda^k=1.$$

$\lambda^k\geq 0$, for all *k*. $x_{i,j,l,t}$, $s_{i,j,l,t}$, $r_{i,j,l,t}$, yr , $yo_t \geq 0$.

Let $\theta_{i,l,t}$ be the dual prices associated with the inventory balance constraint (3.5) for the restricted master, and let λ^k be associated with the convexity constraints for the subproblem. Evaluating the columns of λ^k, we obtain the reduced costs.

Step 0: Initialize. Set iteration $k = 1$ and set an initial feasible solution $(w, wr, f, z, q)^0$.

Step 1: Solve the master problem and obtain solution $\lambda 1^1$ and dual price $\theta^1_{i,l,t}$.

Step 2: Solve the trawler scheduling subproblem $(PR1_\theta)$. Obtain $(w, wr, f, z, q)^1$ which becomes coefficient for a new variable $\lambda 1^2$.

Step 3: If master profit equals subproblem profit, i.e., if $v(M^k) = v(S^k)$ then STOP. Else go back to step 1.

Using the modified DWD for the IFPM, we solve the same 5-period model as we solved in Section 5.5 and obtained the same profit. It takes only 245 iterations and 20 minutes to solve the 5-period model. The results are shown in Table 5.8.

	DWD (5-period)	Modified DWD (5-period)	Modified DWD (10- period)
Number of Iterations	1168	245	1311
Computational time	3:58:49	20:35	5:51:19
Subproblem1 profit	$432,138	$522,764	$881,048
Subproblem2 profit	$90628	-	-
Total profit	$522,763.5	$522,763.5	$1066,350
Master profit	$522,763.5	$522,763.5	$1066,350

Table 5.8: Comparison of iterations, and computation time to solve a 5 and a10-period models by DWD & modified DWD.

With the DWD algorithm, solution of a 5-period model took around four hours and 1,068 iterations. The modified DWD takes only 20 minutes and 245 iterations. The 10-period model, which was abandoned under the previous DWD method, takes 1311 iterations to solve by modified DWD method. So, the modified DWD is more effective to solve the IFPM. But it is still time consuming and not efficient to solve higher period problems. That is why we continue to find a better and efficient solution procedure of the IFPM.

In the following section, we develop a DBP method for efficient solution of the IFPM.

5.7 Decomposition-based pricing

In this section, we apply decomposition-based pricing (DBP) for the efficient solution of the IFPM. Mamer and McBride (2000) developed DBP for a multi-commodity LP problem. Carvalho (1998) used DBP in a column generation and branch-and-bound method for cutting stock problem for the general integer variables, not restricted to be binary. Raffensperger and Schrage (2006) used DBP with a scheduling model for a tank battalion. We apply DBP for solving our mixed integer IFPM. In DBP, the subproblems are identical to the subproblems of DWD, but the DWD master is replaced by a version of the original problem with all of the original rows and a subset of original columns, termed the restricted master. As with DWD, subproblems are created by dualizing a subset of the constraints, and these subproblems are identical to $S_1^k = PR1_\theta$ and $S_2^k = PR2_\theta$ from the DWD.

In DBP, instead of using the subproblem to produce an extreme point of the relaxed polytope for inclusion in a restricted master problem, we include the optimal basic columns of the subproblems into the restricted master. We then solve this restricted master to obtain an improved primal basic feasible solution to the original problem, and to obtain new dual prices. We then pass the dual solutions obtained from this restricted master to the subproblems. The procedure terminates when no positive variables entered into the restricted master or when the objective value of the subproblems and that of the restricted master are equal. Constructing the restricted problem in this fashion assures a basic feasible solution to the original problem, and the size of the restricted master tends to be small.

The IFPM is a MILP model, so we cannot guarantee strong duality (outside of a custom branch and bound algorithm). Hence this is a heuristic method. However, through careful choice of initial feasible solution and stopping criteria, we obtain excellent bounds. And the solution times obtained are faster than the direct solutions with CPLEX.

5.7.1 DBP for the IFPM

Using LR, we first relax the inventory balance constraint (3.5) as in Section 5.4. Let $\theta_{i,l,t}$ be the simplex multipliers associated with the inventory balance constraint (3.5) in the restricted master. We define the restricted master as the original problem for the IFPM, but restricted to a smaller set of variables I^k. Set I^k is the set of all positive variables in the master at iteration *k*. Set I^k increases in size with each iteration, because each iteration of the subproblems adds new positive variables to I^k. Computationally, we found that the number of variables in I^k at any iteration is much less than the number of variables in the original problem. We then define the restricted master as follows.

Restricted master (M^k)**:**

Maximize

$$= -\left(\sum_p \sum_a \sum_u \sum_t \sum_v (t-u) V_{t,v} w_{p,a,u,t,v} + \sum_i \sum_l \sum_t \sum_v \sum_a C_{a,i,t,v} f_{a,i,l,t,v} + \sum_i \sum_l \sum_t I_t z_{i,l,t} \right)$$
$$+ \left(\sum_i \sum_j \sum_l \sum_t P_{i,j,l} s_{i,j,l,t} - \sum_t Lr_t\, yr - \sum_t Lo\, yo_t - \sum_i \sum_j \sum_l \sum_t J_t r_{i,j,l,t} \right).$$

Subject to 3.1 – 3.4 and 3.5 – 3.9, 3.13, 4.1 – 4.3.

$f_{a,i,l,t,v}$, $w_{p,a,u,t,v}$, $wr_{t,v}$, $q_{a,i,t} \geq 0$ and $w_{p,a,u,t,v} \in \{0,1\}$, $wr_{t,v} \in \{0,1\}$.

$x_{i,j,l,t}$, $s_{i,j,l,t}$, $r_{i,j,l,t}$, yr, yo_t, $\geq 0. \in I^k$, where $I^k = \{i : x^i > 0\}$.

In matrix-vector notation, M^k can be written as

(M^k) Maximize $c^1 w + c^2 f + c^3 x$,

subject to constraint sets (5.1) to (5.4),

$f \geq 0$, $w \in \{0,1\}$, $x \geq 0$, with $f, w, x \in I^k$, here I^k is the index set of all positive variables $f, w, x \geq 0$.

To calculate the dual prices, we use LP relaxation and solve the trawler scheduling subproblem $PR1_\theta$ and the processing subproblem $PR2_\theta$. We solve the restricted master as an LP and pass the new dual prices to the subproblems. When the optimal values of the subproblem and the restricted master are equal, then stop. The obtained restricted master problem is then solved as an IP. In the following section, we present the DBP algorithm for solving the IFPM.

5.7.2 DBP algorithm for the IFPM

Our DBP is summarised as the following algorithm.

Step 0: Initialize. Set iteration $k = 1$. Pick a set of prices $\theta^k_{i,l,t}$. We used three alternate methods to pick an initial set of prices θ^1.

I1: Start with $\theta^1 = 0$.

I2: Start with θ^1 as the dual prices from the relaxed constraints of the IFPM LP relaxation.

I3: Start with heuristic dual prices, $\theta^1{}_{ilt} = -\sum_{j:Fij>0} P_{ij}/(2.5\cdot F_{ij})$, where $F_{i,j}$ is the fillet percentage of raw material and $P_{i,j,l}$ is the profit of processing product j of quality l from raw materials i.

Step1: Use LP Relaxation and solve subproblem $PR1_\theta$ and solve subproblem $PR2_\theta$.

For $x^i > 0$, put i in I^k, where $I^k = \{i : x^i > 0$ in $PR1_\theta$ and $PR2_\theta$ for any iteration $1,2,\ldots,K\}$.

Step 2: Solve M^k and get dual prices $\theta^k_{i,l,t}$.

Step 3: For stopping criterion, we used two alternate methods:

SC1: Stop when $v(PR1_\theta + PR2_\theta) = v(M^{k+1})$. Here we solve the trawler scheduling problem as an LP. By solving this subproblem as an LP, we find good variables to add to the restricted master, with fast computation time.

SC2: Stop when no new variables enter into the restricted master. Here we solve the trawler scheduling problem as an IP.

Else go to step 1.

Step 4: After the LP optimum is found, solve the final restricted master problem as an IP.

We present a flowchart in Figure 5.1 to show the schematic steps of our DBP method for solving our IFPM.

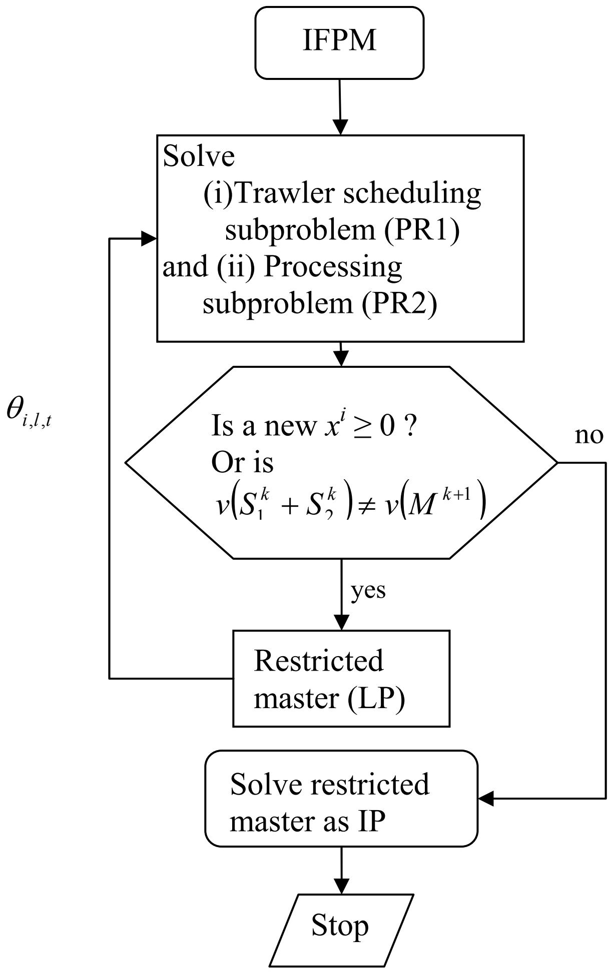

Figure 5.1: Flowchart of DBP

Before reporting the DBP solutions, we present the number of variables, LP relaxation solutions and IP solutions of 5, 10, 15, 20, 25, and 30-period models in Table 5.9a.

Planning horizon	Solution time(sec)	Number of variables in primal	LP relaxation solution	IP solution	% solution gap
5	3	2,193	$522,764	$522,764	0
10	5	4,423	$1,066,350	$1,065,775	0.05
15	17	6,803	$1,607,944	$1,582,008	1.61
20	131	9,333	$1,898,411	$1,880,196	0.96
25	22	11,989	$2,141,757	$2,121,887	0.93
30	18,620	16,139	$2,331,037	$2,300,871	1.29

Table 5.9a: LP relaxation solution and IP solution of different planning horizon models

Depending on the initial feasible solution and stopping criterion, we ran the DBP algorithm in five different ways: I1-SC1, I1-SC2, I2-SC1, I2-SC2, I3-SC1, shown in Table 5.9b. We also calculate the percentage solution gap as

$100 \times$ (IP solution – DBP solution)/ IP solution.

Solution method	Planning horizon	Iterations	Seconds solution time	Variables in final master	DBP solution ($)	% solution gap
I1-SC1	5	26	156	1,308	522,764	0.00%
I1-SC1	10	29	257	2,815	1,065,775	0.00%
I1-SC1	15	32	341	4,272	1,579,440	0.16%
I1-SC1	20	29	365	5,691	1,874,097	0.32%
I1-SC1	25	29	414	7,026	2,119,938	0.09%
I1-SC1	30	25	544	8,115	2,293,803	0.31%
I1-SC2	5	29	211	1,252	522,764	0.00%
I1-SC2	10	30	258	2,576	1,065,538	0.02%
I1-SC2	15	32	335	3,881	1,579,309	0.17%
I1-SC2	20	27	348	5,065	1,870,047	0.54%
I1-SC2	25	29	557	6,253	2,118,528	0.16%
I1-SC2	30	31	1,737	7,324	2,288,997	0.52%
I2-SC1	5	27	192	1,356	522,764	0.00%
I2-SC1	10	33	292	2,873	1,065,531	0.02%
I2-SC1	15	30	322	4,378	1,579,321	0.17%
I2-SC1	20	28	496	5,874	1,864,368	0.84%
I2-SC1	25	27	433	7,135	2,117,990	0.18%
I2-SC1	30	32	1,042	8,277	2,266,274	1.50%
I2-SC2	5	28	208	1,282	522,764	0.00%
I2-SC2	10	28	252	2,724	1,065,712	0.01%
I2-SC2	15	35	373	4,092	1,579,466	0.16%
I2-SC2	20	29	359	5,420	1,875,597	0.24%
I2-SC2	25	35	534	6,540	2,111,616	0.48%
I2-SC2	30	30	650	7,623	2,292,894	0.35%
I3-SC1	5	26	178	1,325	522,764	0.00%
I3-SC1	10	32	275	2,784	1,065,775	0.00%
I3-SC1	15	30	312	4,130	1,579,447	0.16%
I3-SC1	20	31	351	5,524	1,876,023	0.22%
I3-SC1	25	32	487	7,135	2,120,282	0.08%
I3-SC1	30	27	613	8,052	2,295,376	0.23%

Table 5.9b. Numerical results for DBP under different initial dual prices and stopping criteria.

From Table 5.9a and Table 5.9b, we observed that the solutions obtained from all the cases of the proposed DBP procedure are very close to the original solution. Also the comparison of these tables show that the classical LR results of improved bounds, when the subproblem is not naturally integral, does not follow analogously for DBP. This is because the master does not produce a convex combination of subproblem solutions. We also notice that, the best average percentage solution gap is only 0.12% and is obtained with I3-SC1. The second best percentage solution gap is only 0.14% obtained with I1-SC1.

Table 5.10 shows that DBP takes fewer iterations and much less time to solve the IFPM than DWD and modified DWD. Figure 5.2 shows that the number of variables in the restricted problem is much less than that of the original problem.

	DWD	Modified DWD	DBP
Number of Iterations	1168	245	26
Computational time	3:58:49	00:20:35	0:02:58
Subproblem1 profit	$432,138.0	$522,763.5	$432,132
Subproblem2 profit	$90,628.5	-	$90,631.5
Total profit	$522,763.5	$522,763.5	$522,763.5
Master profit	$522,763.5	$522,763.5	$522,763.5

Table 5.10: Comparison of the number of iterations, and computational time taken by different methods to solve a 5-period model.

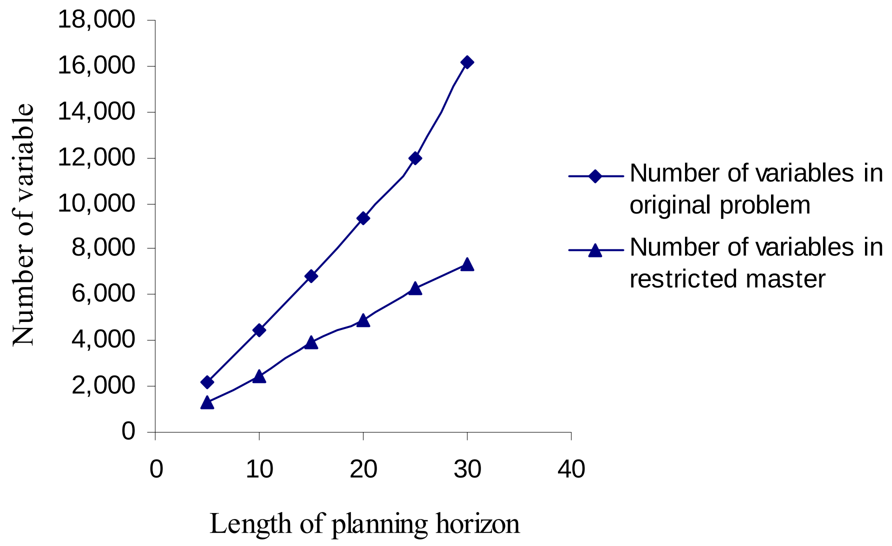

Figure 5.2: Comparison of the number of decision variables in DBP to that of IP.

Since we obtained I1-SC1 and I1-SC3 as the best two criterions from the experiments with the original problem, we will test the other different problems IFPMS, IFPML, and IFPMXL using these two criteria. Results are shown in Table 5.11.

Problem	Solution method	Iterations	Seconds solution time	Variables in final master	DBP solution ($)	% solution gap
IFPMS	I3-SC1	18	301	3,901	1,821,780	2.2
	I1-SC1	15	442	3,871	1,842,726	1.1
IFPML	I1-SC1	30	1,946	8,933	2,547,730	0.1
	I3-SC1	34	1,245	7,904	2,545,035	0.2
IFPMXL	I1-SC1	28	1,026	10,698	2,539,377	0.2
	I3-SC1	29	1,039	10,585	2,538,542	0.3

Table 5.11. Numerical results for DBP under different initial dual prices and stopping criteria for a 30-period IFPMS, IFPML, and IFPMXL.

From Table 5.11, we notice IFPMS has higher solution gap than the other examples. To find why this gap is, we investigate the scheduling of the two trawlers used in this example. We observe two idle periods when trawler stay at port in the DBP method while in the direct CPLEX solution there is one idle period. As a result the DBP method landed fewer amounts of raw materials than that of the direct method. So the processing schedule changes accordingly. Hence there is a little high solution gap. In the other example we notice same number of trawler trips in the DBP procedure and direct CPLEX solution. But the landing periods are little different. Hence the DBP got a very small solution gap.

From all four different problem instances, we found that our DBP produced excellent results and took less time for solving the different planning horizon problems. The solution gap was very small.

5.8 Conclusion

In this chapter, we described our work with relaxation and decomposition techniques for solving the IFPM. We found that the LR with subgradient optimization method is ineffective for solving the IFPM because of its unpredictable convergence behaviour and lack of appropriate stopping criteria. DWD is also ineffective. Finally we proposed a new decomposition-based pricing procedure for solving the large IFPM. From the experiments with four different problem instances, we found that our DBP procedure for the IFPM is the most effective method by far. It gives excellent computation times. Numerical results for several planning horizon models of four different problem instances were presented.

This method also was foreshadowed in the computational experiments with some more challenging data sets conducted in Chapter 4. The results are consistent with those found in this chapter.

In the following chapter, we develop a new method which we call decomposition-based O'Neill pricing (DBONP), to try to improve the solutions obtained from this chapter.

Chapter 6

Solution of IFPM with Decomposition-Based O'Neill Pricing

6.1 Introduction

In Chapter 5, we developed a decomposition-based pricing (DBP) to solve the IFPM. The results were close to the optimal solutions. In this chapter, we propose an algorithm to improve the optimal value obtained by the DBP procedure proposed in Chapter 5. We name the proposed technique as *decomposition-based O'Neill pricing* (DBONP). The DBONP is based on the theorem of Gomory & Baumol (1960), O' Neill et al (2005), and decomposition-based pricing.

The remainder of this chapter is organized as follows. In Section 6.2, we review O'Neill's pricing method. In Section 6.3, we discuss the mathematical formulation of the proposed DBONP method. In Section 6.4, we present the DBONP algorithm along with numerical examples.

6.2 O'Neill pricing method

Based on the theorem of Gomory and Baumol (1960), O'Neill *et. al* (2005) developed a technique for constructing a set of linear prices from solving a MILP and an associated LP. They first solved a MILP, set the integer variables to their optimal values, and then removed the integrality constraints to convert the MILP to an LP. They used the dual prices obtained from this LP to form an efficient contract (the dual of the resulting LP).

We reproduce Gomory & Baumol (1960)'s theorem here for completeness.

Theorem 6.1: A MILP with *m* continuous variables and *n* integer variables $\left(R^m \times Z^n\right)$ that has a feasible and bounded optimal solution can be converted to an LP with at most ($m + n$) continuous variables R^{m+n} and at most *n* additional linear constraints.

We now describe O'Neill's method using our IFPM as follows.

Step 1. O'Neill et al. (2005) first solve the *MILP*.

(MILP) maximize $c^1 w + c^2 f + c^3 x$, subject to

$$A^0 w + f = 0. \tag{6.1}$$

$$D^1 w = b^1. \tag{6.2}$$

$$D^2 x = b^2. \tag{6.3}$$

$$A^1 f + A^2 x = b^0. \tag{6.4}$$

$$w \in \{0,1\} \tag{6.5a}$$

$$f,\ x \geq 0. \tag{6.5b}$$

Step 2. For each integer variable, set $w = w^*$, its value at the end of Step 1 and remove the integrality constraints (6.5a). The resulting LP model is as follows.

$$
\begin{array}{llllll}
\textit{(LMILP)} & \text{maximize} & c^1 w + c^2 f + c^3 x, \text{ subject to} & & \\
& & A^0 w + f & = 0. & (6.6) \\
& & D^1 w & = b^1. & (6.7) \\
& & D^2 x & = b^2. & (6.8) \\
& & A^1 f + A^2 x & = b^0. & (6.9) \\
& & w = w^* & & (6.10) \\
& & w,\ f,\ x \geq 0. & & (6.11)
\end{array}
$$

Step 3. Calculate the dual prices by solving *LMILP* to form an efficient contract (the dual of *LMILP*) for solving the original *MILP*.

We now require the following theorem to establish the relationship between the value of the optimal solution of *MILP*, and that of *LMILP*. The theorem is contained in O' Neill *et. al* (2005). We reproduce it here.

Theorem 6.2: $v(MILP) = v(LMILP)$. Proof can be found in O' Neill *et. al* (2005).

O'Neill et all (2005), showed that the optimal solution to an LP that solves the MILP has the dual variables that have the traditional interpretation as prices, explicitly prices integral activities, and clear the market in the performance of nonconvexities. For this, they first solve the MILP as discussed above, remove the integrality restriction and insert equality constraints that force the integer variables to assume their optimal values in the resulting LP. The authors then solved the LP to find the associated dual prices on the market clearing conditions and added equality constraints. These dual prices are then used as prices to support a competitive equilibrium.

In our DBONP, we have one trawler scheduling integer subproblem, and one processing LP subproblem. For the trawler scheduling subproblem, we apply O'Neill et al (2005) concept. We apply DBP to obtain the final restricted master, which we use as the initial problem in the DBONP approach. We solve this master as an IP and set an equation for each integer variable, remove the integrality restriction, and solve the restricted master as an LP. In this way, we are able to bring more good variables in the restricted master and obtain better dual prices to solve the subproblems. In the following section, we describe the mathematical formulation of the DBONP for solving the IFPM.

6.3 Decomposition-based O'Neill pricing (DBONP)

In this section, we present the decomposition-based O'Neill pricing (DBONP) method for the solution of the IFPM. The DBONP has two loops. Loop1 uses decomposition-based pricing as in Chapter 5 to get the final restricted master. In loop2, we solve the final restricted master as an IP, we set the integer variables to their optimal values, and convert the restricted master to an LP. Then we solve this LP master to obtain new dual prices, and use the dual prices to solve subproblems. This procedure terminates when no new variable is found.

Loop 1. We first relax the inventory balance constraint (3.5), and then apply the decomposition-based pricing algorithm developed in Chapter 5, to obtain the final restricted master as an LP.

Loop 2. We solve this final restricted master as an IP, set the integer variables to their optimal values, and convert it to an LP. We then calculate the dual prices associated with the relaxed inventory balance (3.5) constraint and the equations associated with the integer variables.

Let $\theta_{i,l,t}$ for all *i*, *l* and *t,* be the dual prices associated with the inventory balance constraint (3.5) and $\theta 1_{p,a,u,t,v}$, and $\theta 2_{t,v}$ be the dual prices associated with the integrality constraints (6.10). Then the trawler scheduling subproblem for the fishery can be presented as:

Trawler scheduling subproblem S_1^k

Maximize

$$= -\left(\sum_p \sum_a \sum_u \sum_t \sum_v (t-u) V_{t,v} w_{p,a,u,t,v} + \sum_i \sum_l \sum_t \sum_v \sum_a C_{a,i,t,v} f_{a,i,l,t,v} + \sum_i \sum_l \sum_t I_t z_{i,l,t} \right)$$
$$- \sum_{i,l,t,k} \theta_{i,l,t} \left(z_{i,l,t-1} - z_{i,l,t} + \sum_a \sum_v f_{a,i,l,t,v} \right)$$
$$- \sum_p \sum_a \sum_u \sum_t \sum_v \theta 1_{p,a,u,t,v} \left(w_{p,a,u,t,v} - w^*_{p,a,u,t,v} \right) - \sum_t \sum_v \theta 2_{t,v} \left(wr_{t,v} - wr^*_{t,v} \right).$$

subject to 3.1 – 3.4, 3.6, 3.13.

$f_{a,i,l,t,v}$, $z_{i,l,t}$, $w_{p,a,u,t,v}$, $wr_{t,v}$, $q_{a,i,t} \geq 0$ and $w_{p,a,u,t,v} \in \{0,1\}$, $wr_{t,v} \in \{0,1\}$.

In matrix vector notation, it can be written as,

$$(S_1^k)\ \underset{f,w,x}{\text{Max}} \left\{ c^1 w + c^2 f - \theta\left(A^1 f - b^0\right) - \theta 1\left(w - w^*\right) \mid A^0 w + f = 0, D^1 w \leq b^1 \right\}.$$

The processing subproblem remains same as in DBP.

The restricted master in the second loop takes the following form.

Restricted Master (M^k):

Maximize

$$= -\left(\sum_p \sum_a \sum_u \sum_t \sum_v (t-u) V_{t,v} w_{p,a,u,t,v} + \sum_i \sum_l \sum_t \sum_v \sum_a C_{a,i,t,v} f_{a,i,l,t,v} + \sum_i \sum_l \sum_t I_t z_{i,l,t} \right)$$
$$+ \left(\sum_i \sum_j \sum_l \sum_t P_{i,j,l} s_{i,j,l,t} - \sum_t Lr_t \, yr - \sum_t Lo \, yo_t - \sum_i \sum_j \sum_l \sum_t J_t r_{i,j,l,t} \right)$$

Subject to 3.1 – 3.4, 3.5 – 3.9, 3.13, 4.1 – 4.3.

For integer variables, set the equations as follows

$$w^k_{p,a,u,t,v} = w^*_{p,a,u,t,v} . \tag{6.12}$$

$$wr^k_{t,v} = wr^*_{t,v} . \tag{6.13}$$

$f_{a,i,l,t,v}$, $w_{p,a,u,t,v}$, $wr_{t,v}$, $q_{a,i,t} \geq 0$.

$x_{i,j,l,t}$, $s_{i,j,l,t}$, $r_{i,j,l,t}$, yr , yo_t, $\geq 0. \in I^k$.

In matrix vector notation, M^k can be written as,

(M^k) Maximize $c^1 w + c^2 f + c^3 x$,

subject to constraint sets (6.1) to (6.4),

$$w = w^*$$

$f, w, x \geq 0$, with $f, w, x \in I^k$, here I^k is the index set of all positive variables $f, w, x \geq 0$.

In the following section, we present the DBONP algorithm for solving the IFPM.

6.4 DBONP algorithm

In this section, we summarise the DBONP algorithm as follows.

LOOP 1

Step 0: Initialize. Set iteration $k = 1$. Choose a set of prices $\theta^k_{i,l,t}$

(let $\theta^1_{i,l,t}$ is zero).

Step1: Solve subproblem S^k_1 and solve subproblem S^k_2. For $x^i > 0$ put i in I^k, where $I^k = \{i : x^i > 0$ in S_1, and S_2 for any iteration $1,2,\ldots,K\}$.

Step 2: Solve the restricted master M^k as LP and get dual prices $\theta^k_{i,l,t}$ and pass them to the subproblems.

Step 3: If $v(S^k_1 + S^k_2) = v(M^{k+1})$, then go to Loop2. Else update k and go to Step 1.

LOOP 2

Step 4: Solve the final restricted master problem obtained from Step 3 as an IP.

Step 5: For integer variables, fix $x^i = x^{i*}$.

Step 6: Solve master with fixed x^i as LP. Obtain dual prices $\theta_{i,l,t}$, $\theta1_{p,a,i,l,t,v}$, $\theta2_{t,v}$ and pass them to the subproblems.

Step 7: Solve the subproblems with the dual prices obtained from step 6. If no new variables enter into the restricted master, then stop. Else go back to step 4.

We present the following flowchart to show the schematic steps of DBONP in Figure 6.1.

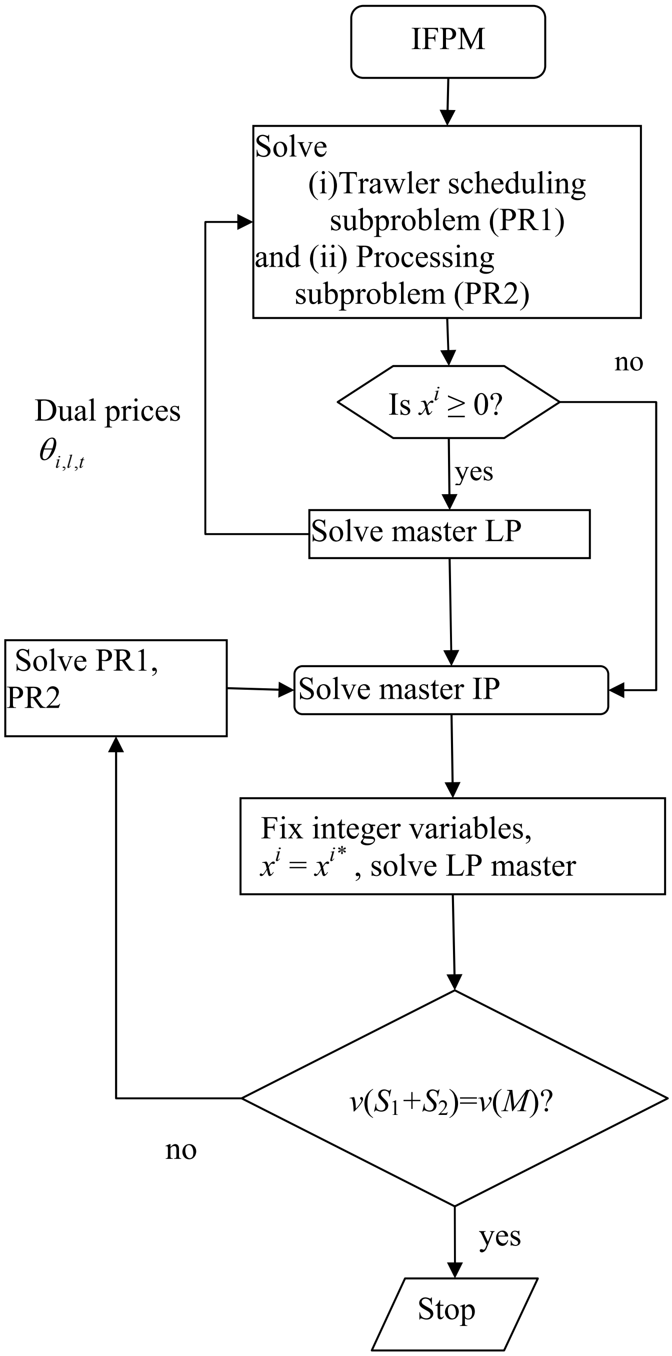

Figure 6.1: Flowchart of DBONP.

6.5 Numerical results

We solve 5 to 30-period models of the original problem using the solution criteria I1-SC1 as in Chapter 5. We compare the DBONP solutions with those obtained from the original IFPM, and DBP. Results are presented in Table 6.1.

We observe no solution gap for 5, 10 and 25-period models. But the 15, 20, and 30-period models have slight gaps. For example, a 30-period model has only 0.02% solution gap. The average percentage solution gap of six different planning horizon models is only 0.04%.

Length of planning Horizon	Number of variables	Number of Iterations	Solution time (s)	DBP solution ($)	DBONP solution ($)	% solution gap
5	489	29	217	522,764	522,764	0.00%
10	1,284	27	216	1,065,540	1,065,775	0.00%
15	2,229	33	345	1,579,309	1,579,570	0.15%
20	3,324	48	912	1,874,097	1,878,580	0.08%
25	6,440	45	796	2,120,282	2,121,887	0.00%
30	6,938	44	3,562	2,293,803	2,300,230	0.02%

Table 6.1: Comparison of the optimal solutions obtained from DBP and DBONP methods using the criterion I1-SC1.

We then solve the same problems using the solution criteria I3-SC1 and run the model varying the length of planning horizon. Results are reported in Table 6.2. Solutions obtained from DBONP are very close to the true optima. The average percentage gap is only 0.06%, but 0.02% worse than the table above.

Planning horizon	Number of Variables	Number of Iterations	Solution time (s)	DBP solution ($)	DBONP solution ($)	% solution gap
5	1,264	29	208	522,764	522,764	0.00%
10	2,601	30	266	1,065,540	1,065,540	0.02%
15	4,087	36	387	1,579,309	1,580,670	0.08%
20	4,926	50	1,045	1,874,097	1,873,950	0.30%
25	6,259	43	710	2,120,282	2,121,887	0.00%
30	8,277	50	3,129	2,293,803	2,300,460	0.01%

Table 6.2: Comparison of the number of iterations, time and solutions obtained from DBP and DBONP.

We also solve three other problem instances IFPMS, IFPML, and IFPMXL using the solution criteria I1-SC1 and I3-SC1 (since these two stopping criterions were the best in Chapter 5). Results are shown in Table 6.3. We notice that all the solutions are very close to the IP optimum. The average solution gap is below 1%.

Problem	Criterion	Number of variables	Number of iterations	Solution time (s)	DBP solution ($)	DBONP solution ($)	IP solution ($)	% solution gap
IFPMS	I1-SC1	3,964	26	2,405	1,821,780	1,863,640	1,874,130	0.5
	I3-SC1	4,012	25	2,227	1,842,726	1,869,970	1,874,130	0.2
IFPML	I1-SC1	8,806	37	2,920	2,547,730	2,550,150	2,550,260	0.004
	I3-SC1	8,094	33	2,537	2,545,035	2,549,040	2,550,260	0.04
IFPMXL	I1-SC1	10,137	32	2,985	2,539,377	2,544,960	2,568,376	0.91
	I3-SC1	10,097	31	2,896	2,538,542	2,544,310	2,568,376	0.93

Table 6.3: Comparison of the optimal solutions of 30-period IFPMS, IFPML, and IFPMXL problems obtained by DBP and DBONP methods using different criterions.

Table 6.1 and 6.2 show that the solutions obtained from DBONP are either equal to the optimal solutions (5-period, 10-period, 25-period models) or very close to the optimal solutions (15-period, 20-period and 30-period models).

To see why this little difference in profit remains, we compare the true optimal solution with that from DBONP. The number of trawler trips in DBONP coincides with that of the original problem. But we notice a little difference in the period of landings. For example, in the original problem of a 30-period model, trawler 1 lands its catch on period 4, 7, 10, 14, 18, 22, 26, and 30. On the other hand, in the DBONP method, in a 30-period model, trawler 1 lands its catch on period 4, 7, 11, 15, 19, 23, 26, and 30. Results are shown in Figure 6.2 and Figure 6.3. As a result, we notice a slight change in the processing accordingly.

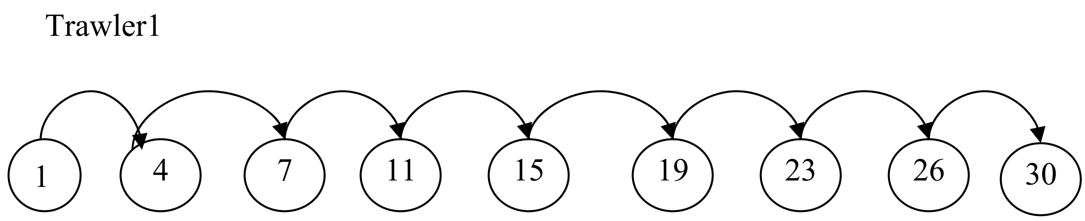

Figure 6. 2: Scheduling of trawler 1 in DBONP

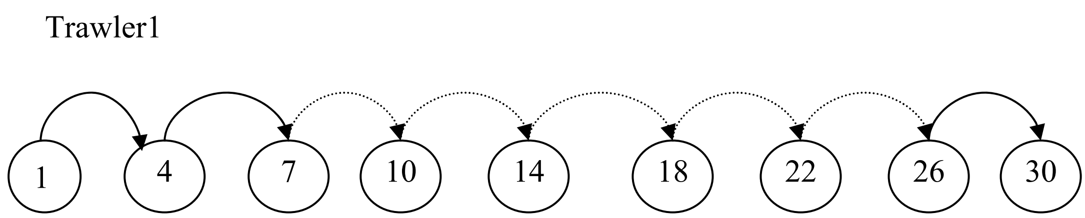

Figure 6.3: Optimal scheduling of trawler 1.

6.6 Comparison of DBP and DBONP

In this section, we compare the number of iterations, computation time, number of variables and optimal values obtained by solving different planning horizon models by DBP and DBONP. From Figure 6.4, 6.5 and 6.6, we observe that DBONP takes a higher number of iterations and higher computation time but produces better solutions than that of DBP.

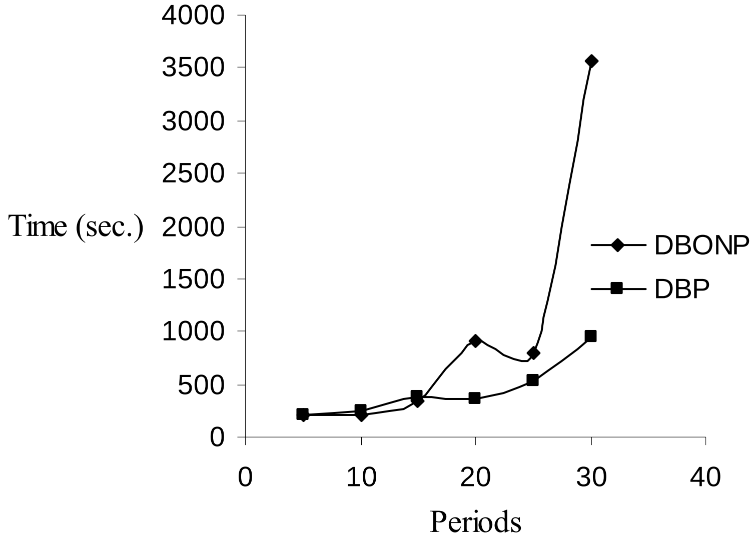

Figure 6. 4: Comparison of solution time required to solve different planning horizons by DBP, DBONP.

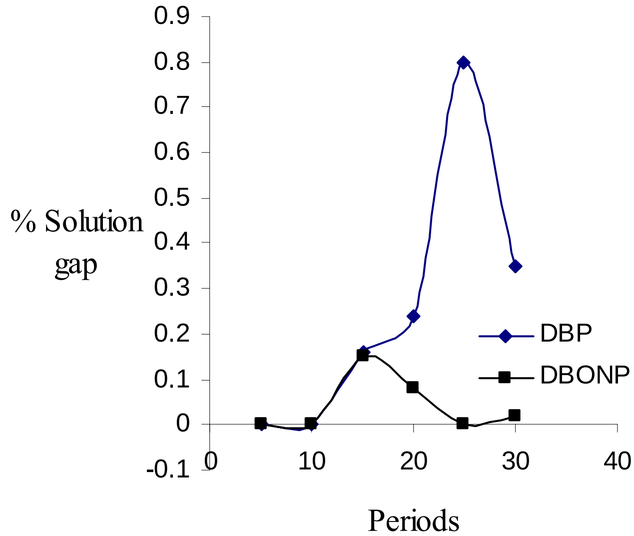

Figure 6. 5: Comparison of percentage solution gap of DBP and DBONP.

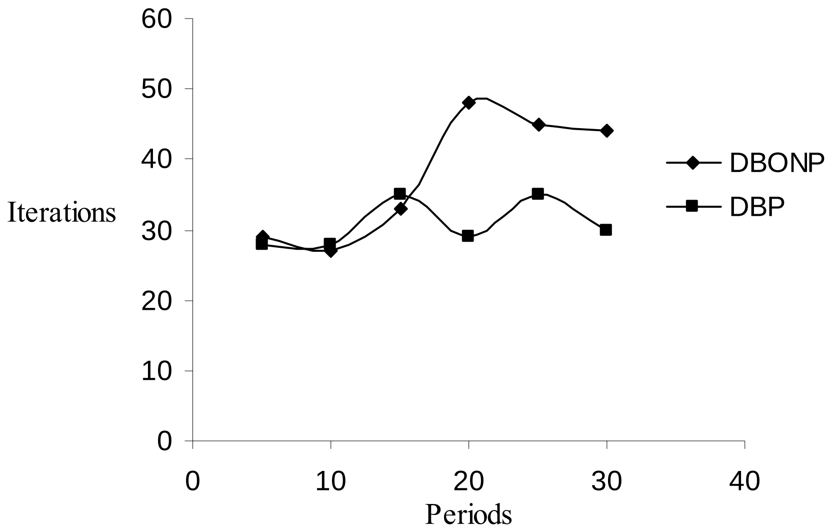

Figure 6. 6: Comparison of number of iterations required to solve different planning horizons by DBP, and DBONP.

6.7 Conclusion

In this chapter, we developed the DBONP algorithm to improve the solutions obtained by the DBP procedure developed in Chapter 5. We incorporated DBP and O'Neill pricing methods to develop our DBONP. Solving models with different planning horizons of different problem instances, we observed that it took shorter time than SO and DWD but took slightly longer time than the DBP. Although the DBONP took longer time, it produced better solutions than DBP. To find more close solution and to reduce the required solution time while obtaining better solution, we will develop a reduced cost-based pricing method in Chapter 7.

Chapter 7

A Reduced Cost-Based Pricing Method for Solving IFPM

7.1 Introduction

In this chapter, we develop a reduced cost-based pricing (RCBP) algorithm for solving the IFPM. In 1985, Martin *et al.* presented a reduced cost-based branch-and-bound method for solving mixed integer programs. The author formulated two candidate problems on the basis of 0 or 1-integer variables and then optimized both of the candidate problems to get the solution of the MILP. Unlike Martin et al (1985), we set constraints for both 0 and 1-integer variables (O'Neill et al, 2005) in the same candidate problem, which is the restricted master in the proposed RCBP method. We also decomposed the mixed integer IFPM into trawler scheduling and processing subproblems. The trawler scheduling subproblem was integer and the processing subproblem was linear. Instead of solving the IP trawler scheduling subproblem, we calculate the reduced cost for each variable to choose which variable(s) to bring into the restricted master. Instead of bringing one variable with most negative reduced cost as in the usual simplex method of Dantzig developed in 1947, we bring all variables with negative reduced cost from the trawler scheduling IP subproblem into the restricted master problem at each iteration. For the LP processing subproblem, we used the DBP to bring the positive variables into the restricted master problem. These provide a faster way to solve our IFPM efficiently.

The remainder of this chapter is organized as follows. In Section 7.2, we present the reduced cost calculation. The processing subproblem and the restricted master are the same as the DBP method developed in Chapter 5. But instead of using the trawler scheduling subproblem, we calculate the reduced cost of the variables of that subproblem. In Section 7.3, we present the RCBP algorithm. Section 7.4 compares the RCBP solutions with those from DBP and DBONP.

7.2 Reduced cost of a variable

The reduced cost of a variable x_j with the objective function coefficient c_j is the net change in the objective function to generate one unit of x_j and is defined by $\bar{c}_j = c_j - z_j$, where $z_j = c_{BV} B^{-1} a_j$. Here c_{BV} is the cost coefficient of basic variables and $B^{-1} a_j$ is the column for x_j in optimal table's constraints. The reduced cost gives the net impact of a variable on the objective function related to the current solution. For a maximization problem, the variable with largest negative reduced cost is the incoming variable. The reduced costs are defined for non-basic variables only. Reduced costs are evaluated with respect to the current basis.

We calculate the reduced cost for all the non-basic variables in the trawler scheduling subproblem as follows. Following AMPL notation, we denote the reduced cost of a variable *x* as *x.rc*.

$$w_{p,a,u,t,v}.rc = -(t-u)V_v - \theta^1_{p,a,u,t,v} - flow_{p,t,v} + flow_{p,u,v} + \sum_i \sum_l landed\_fish_{p,a,i,l,t,v}\ ET_{a,u,t,v}\ FR_{i,l}\ FRaw_{a,i,v}$$ for all *p, a, u ,t,* and *v*.

$$w_{p,a,1,t,v}.rc = -trawl\_start_{p,v} + flow_{p,t,v}$$ for all *p, a, u* = 1, *t,* and *v*.

Here $\theta^1_{p,a,u,t,v}$, $flow_{p,t,v}$, $flow_{p,u,v}$, $landed\_fish_{p,a,i,l,t,v}$ and $trawl\_start_{p,v}$ are the dual prices for the integrality constraint $w_{p,a,u,t,v} = w^*_{p,a,u,t,v}$, flow (3.3) , trawler start (3.2), and the landed fish (3.1) constraints respectively.

$$wr_{t,v}.rc = -trawl\_start_{p,v}$$ for all *t* =1, *p*, and *v*.

$$wr_{t,v}.rc = -\theta 2_{t,v} + \sum_p flow_{p,t,v}$$ for all *t* = *2…T*, and *v*.

$$wr_{t,v}.rc = -\sum_p flow_{p,t+1,v}$$ for all $1 \le t \le T-1$, and *v*.

Here $\theta 2_{t,v}$, $flow_{p,t,v}$, and $trawl\_start_{p,v}$ are the dual prices for the integrality constraint $wr_{t,v} = wr^*_{t,v}$, flow (3.3) , and trawler start (3.2) constraints respectively.

$$f_{a,i,l,t,v}.rc = -C_i - \theta_{i,l,t} - \sum_p landed\_fish_{p,a,i,l,t,v} + quota\_balance_{a,i,t}$$ for all *a,i,l,t,v*.

Here $\theta_{i,l,t}$, $landed\_fish_{p,a,i,l,t,v}$, and $quota\_balance_{a,i,t}$ are the dual prices for the inventory balance (3.5), landed fish (3.1) and the quota balance (3.13) constraints respectively.

$$z_{i,l,0}.rc = -end\_effect_{i,l}$$ for all *i, l.*

$$z_{i,l,t}.rc = -\theta_{i,l,t+1}$$ for all $1 \le t \le T-1$, *i, l.*

$$z_{i,l,t}.rc = -I + \theta_{i,l,t} - store_t$$ for all *i, l, t.*

$z_{i,l,T}.rc = end\_effect_{i,l}$ for all *i, l.*

Here $end\_effect_{i,l}$, $\theta_{i,l,t}$ and $store_t$ are the dual prices for the end effect (5.1), inventory balance (3.5) and the store (3.6) constraints respectively.

$q_{a,i,0}.rc = initial\_quota_{a,i}$ for all *a, i.*

$q_{a,i,t}.rc = -quota\_balance_{a,i,t+1}$ for all a, i, $1 \le t \le T-1$

$q_{a,i,t}.rc = quota\_balance_{a,i,t}$ for all *a, i, t.*

Here $initial\_quota_{a,i}$, and $quota\_balance_{a,i,t}$ are the dual prices for the initial quota ($q_{a,i,0} = Q_{a,i}$) and the quota balance (3.13) constraints respectively. In the following section, we present the RCBP algorithm for solving the IFPM.

7.3 RCBP algorithm

In this section, we summarise the RCBP algorithm as follows.

Step 0. Initialize. Solve $\overline{P}$ and save its positive variables.

Step 1. Solve the restricted master as an IP and write constraints for all integer variables (O'Neill et al, 2005) as:

$$w_{p,a,u,t,v} = w^{*}_{p,a,u,t,v} \quad \text{for all } p, a, u, t, \text{ and } v. \tag{7.1}$$

$$wr_{t,v} = wr^{*}_{t,v} \quad \text{for all } t, \text{ and } v. \tag{7.2}$$

Step 2a. Solve the restricted master obtained from step 1 as an LP.

Step 2b. Calculate and save dual prices for all the trawler scheduling constraints (3.1 – 3.4, 3.6, 3.13).

Step 3a. For the trawler scheduling constraints, 3.1 – 3.4, 3.6, 3.13, check the reduced costs for all trawler scheduling variables. Add variables with negative reduced cost to the restricted master. This calculation for the trawler scheduling problem is accomplished by one of two following options.

Option 1: All continuous variables appear directly in every restricted master. We bring the only the integer variables with negative reduced cost for fast computation time.

Option 2: Continuous and integer variables with negative reduced cost are added to the restricted master at each iteration.

We use two options for bringing the variables with negative reduced cost from the trawler scheduling subproblem into the restricted master problem. In Option 1, we bring only the integer variables with negative reduced cost from the trawler scheduling subproblem into the master; other continuous variables in that subproblem appear in the master directly. By this option, we find good variables to add the restricted master problem with fast computation time. In Option 2, we bring all variables (integer and continuous) with negative reduced cost into the master.

Step 3b. For the processing subproblem, solve the processing LP subproblem, and add all positive variables to the restricted master as with DBP (Mamer and McBride, 2000).

Step 4. If no new variable enters the restricted master, then stop. Else go back to step 1.

We present a flowchart to show the schematic steps of the RCBP approach in Figure 7.1.

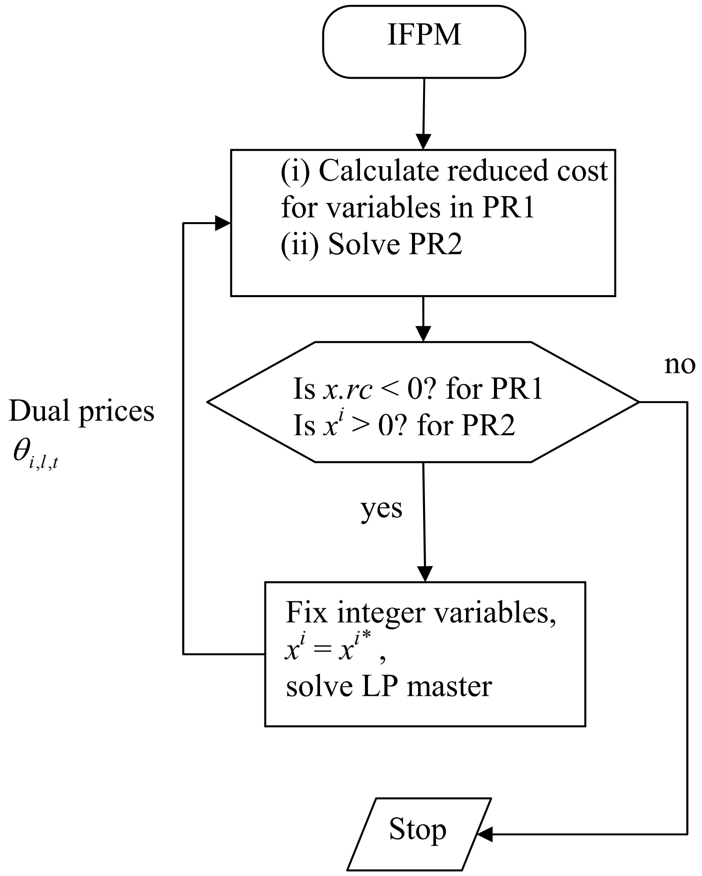

Figure 7.1: Flowchart of RCBP.

7.3.1 Numerical results

We solve different planning horizon models using each option in Step 3a. Option 1 takes fewer iterations and less time to solve the fishery model than does option 2. Results are reported in Table 7.1.

Planning Horizon	Description of entering variables	Number of iterations	Solution time (Sec.)	RCBP optimal value($)	% Solution gap
5	Option 1	3	5	522,764.0	0
	Option 2	5	39	522,764.0	0
10	Option 1	5	15	1,065,538.0	0.02
	Option 2	10	142	1,065,538.0	0.02
15	Option 1	5	53	1,582,008.4	0
	Option 2	11	113	1,582,006.9	0
20	Option 1	4	71	1,879,928.0	0.01
	Option 2	7	109	1,877,275.0	0.15
25	Option 1	8	111	2,121,887.0	0
	Option 2	6	74	2,107,736.0	0.66
30	Option 1	10	901	2,299,648.0	0.05
	Option 2	8	262	2,284,545.0	0.71

Table 7.1: Total profit, iterations, and solution time from RCBP procedure.

7.3.2 Catch rate generation

We thought that the identical average expected catch rate would result in degeneracy. Because, the trawler schedules typically were sequences of 3-period or 4-period trips, e.g., 4, 4, 4, 3 or 3, 4, 4, 4. These two solutions would have identical trawler costs. If the master has 4, 4, 4, 3, then it might have the best solution, but it might get the subproblem to also find 3, 4, 4, 4, which is just an alternate optimum and therefore a wasted iteration. By changing the cost slightly, one solution would be strictly better than the other, and the algorithm not would need to go looking for alternate optima. So, to avoid the degeneracy, instead of using the same amount of average expected catch every period, we changed the catch rate parameter slightly to generate catch rate using the formula

Catch rate = average expected catch + round (average expected catch * 0.0002 * (Rand()-Rand()), 2)

$= E_{a,i,t,v}$ + Round ($E_{a,i,t,v}$ * 0.0002* Rand() – Rand()), 2).

For example, the average expected catch per period of trawler 1 in area 3 for dory is 25,000 kg. But applying the catch generation formula, we generate a slightly different catch rate of trawler 1 for fish species dory in area 3 as 25036.1871 for period 1 and 25002.2059 for period 2. Sample data for 2 periods is shown in Table 7.2.

Species	Dory	Ling	Red cod	Roughy
Average catch per period (Kg)	25,000	500	5000	15000
Generated catch rate for period 1 (Kg)	25036.1871	500.7054	4999.5689	15005.9554
Generated catch rate for period 2 (Kg)	25002.2059	500.0324	4997.2244	15011.6632

Table 7. 2: Sample catch rate generation for trawler 1 in area 3.

Using the created catch rates we solve 5, 10, 15, 20, 25 and 30-period IP and LP relaxation models. The total profit obtained from the models is reported along with solution time and number of variables in Table 7.3. With the generated catch rate, a computer with an Intel Pentium III processor with a clock speed of 665 MHz and 384 MB of RAM, took 476, 26 seconds (over 13 hours) to solve a 30-period model which we failed to solve optimally (terminated after 28 hours) with the original catch rate. We observe that the tiny change in the catch rate makes only a slight change in the solution process. Since this tiny change in the catch rate does not have any significant effect on the solution time, we conclude that the degeneracy is not an issue.

Length of planning horizon	Solution time (sec)	Profit form IP solution ($)	Profit from LP relaxation ($)	% Duality gap	Variables	
					Integer	Continuous
5	3	441,140	441,140	0	75	1422
10	4	926,672	927,313	0.05	300	2957
15	5	1,523,265	1,523,336	0.005	675	4492
20	100	1,794,012	1,825,764	1.73	1200	6027
25	122	2,038,580	2,062,369	1.15	1851	7562
30	476,26	2,207,449	2,246,514	1.73	2556	9097

Table 7.3: IP profit, computational time, and number of variables obtained from the different planning horizons along with LP relaxation profit.

Option 1: Only integer variables with negative reduced cost are added to the restricted master. All continuous variables appear in every restricted master directly.

Length of planning horizon	Iterations	Solution time (sec)	Profit form RCBP solution ($)	Variables	
				Integer	Continuous
5	3	23	441,140	58	893
10	3	27	926,672	182	1,866
15	6	56	1,523,265	344	2,932
20	6	109	1,793,838	670	3,463
25	5	86	2,016,685	979	4,196
30	7	960	2,180,239	1,812	4875

Table 7.4: Total profit, iterations, computation time, and number of integer and continuous variables obtained from different planning horizons by RCBP with option 1.

Option 2: Integer and continuous variables with negative reduced cost are added to the restricted master at each iteration.

Length of planning horizon	Number of Iterations	Solution time (sec)	Profit form RCBP solution ($)	Variables	
				Integer	Continuous
5	3	23	441,140	53	651
10	6	51	926,672	153	1448
15	5	45	1,523,265	287	2222
20	4	64	1,793,277	603	3040
25	6	98	2,036,544	906	3988
30	6	3675	2,200,055	1359	4819

Table 7.4: Total profit, iterations, computation time, and number of integer and continuous variables obtained from different planning horizons by RCBP with option 2.

From the above experiments, we noticed that the tiny change in the catch rate does not make any significant change in the solution process.

7.3.3 Experiments with IFPMS, IFPML, and IFPMXL

In this section, we experiment the workability of the RCBP method with three other different problems IFPMS, IFPML, and IFPMXL. We notice that our RCBP can solve these problems very efficiently with solution very close to the optimal and with less solution time. The results are shown in the Table 7.6. To observe the difference between the IP solution and RCBP solution, we compare the 30-period IFPMS, IFPML, and IFPMXL solutions with direct IP solutions. We notice that the direct IP solution of the 30-period IFPMS has one idle period of trawler 2 but the RCBP solution has two periods (one idle period for trawler 1 and one idle period for trawler 2). As a result the RCBP solution landed (1,056,020 – 1,023,520) = 32,500 kg. less raw materials resulting in a 0.9% less profit. Similarly a 30-period IFPML model landed only 4950 kilogram less fish in RCBP solution than that of the direct IP solution. So the solution gap is only 0.02%. We notice that a 30-period IFPMXL model landed 3440 kilogram less fish in RCBP solution than that of the direct IP solution. As a result the solution gap is only 0.01%.

Problem	Criterion	Number of Iterations	Solution times (s)	RCBP solution ($)	% solution gap
IFPMS	Option 1	8	178	1,857,244	0.9
	Option 2	9	402	1,841,543	1.73
IFPML	Option 1	6	90	2,542,620	0.29
	Option 2	7	381	2,549,627	0.02
IFPMXL	Option 1	10	527	2,538,437	0.32
	Option 2	10	816	2,546,648	0.01

Table 7.6: Total profit, iterations, computation time obtained for a 30-period IFPMS, IFPML, and IFPMXL by RCBP.

7.4 Comparison of DBP, DBONP and RCBP

We present a comparison of the optimal solutions, number of iterations, and solution times obtained from decomposition -based pricing (DBP), decomposition-based O'Neill's pricing (DBONP) and reduced cost-based pricing (RCBP) in Figures 7.2 to 7.4. We observed that the RCBP is the best among the methods we developed. It takes shorter time to solve, fewer iterations and yields better solutions.

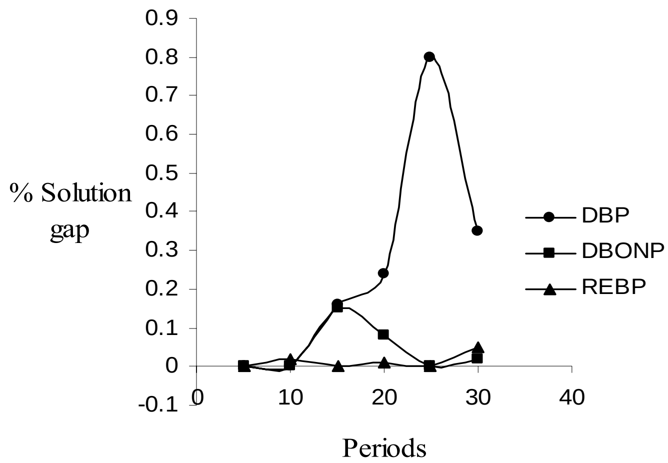

Figure 7.2: Comparison of percentage solution gap of DBP, DBONP, and RCBP.

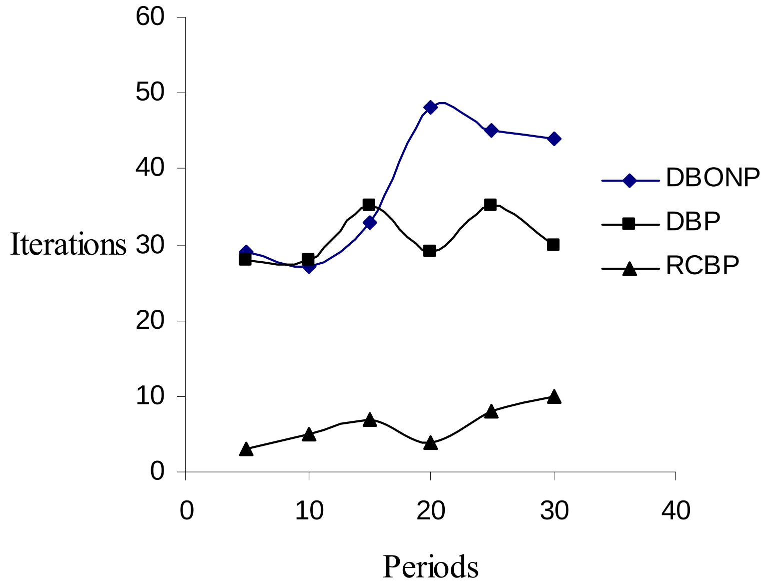

Figure 7.3: Comparison of number of iterations required to solve DBP, DBONP, and RCBP.

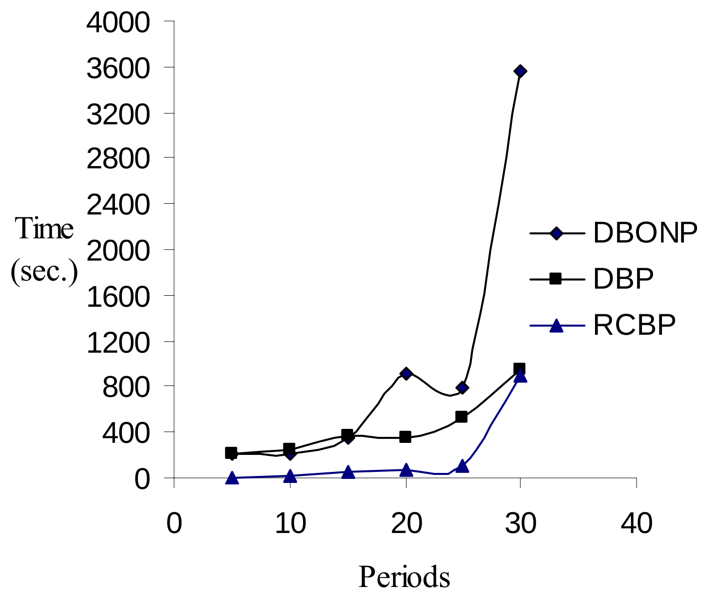

Figure 7. 4: Comparison of solution time required to solve DBP, DBONP, and RCBP.

7.5 Conclusion

In this chapter, we developed a reduced cost-based pricing (RCBP) algorithm for the efficient solution of IFPM. This algorithm is new and could be applied to other IPs. Our focus was the fishery, so other types of models were outside the scope of this thesis, but that could be a good future work. The RCBP approach bears similarities to other work, e.g., simplex method, DWD, and DBP, but those are all for linear programs, while our RCBP is intended for IPs. This new algorithm uses result from the O'Neill et al (2005) paper which is only about pricing an optimal solution for a market with non-convexities. We solved easy LP subproblems and brought positive variables into the restricted master. Instead of solving the IP trawler scheduling subproblem, we calculated the reduced cost for each variable, choosing the variables with negative reduced cost to bring into the restricted master. Solving different planning horizon models of four different problems, we discovered that this method takes shorter time and gives better solutions than DBP and in some cases than DBONP methods developed in Chapter 5 and Chapter 6 respectively.

This method also was foreshadowed in the computational experiments with some more challenging data sets conducted in Chapter 4. The results are consistent with those found in this chapter.

Chapter 8

Contributions and Conclusions

The contributions of this thesis are two fold:

(i) to develop a mathematical framework to assist the fishery to address the planning decisions.

(ii) to find efficient solution procedures of the IFPM.

We developed a mixed integer integrated fishery planning model (IFPM) in Chapter 3 to assist the fishery to address the planning decisions. The model co-ordinates trawler scheduling, fishing, catch quota allocations, processing and labour allocations. The output of the model suggests when and where a trawler should go for fishing, how much raw material should be landed, what amount of product should be produced and how many regular and overtime labour hours are required. To our knowledge, this is the first model to integrate trawler scheduling, production scheduling, and labour scheduling for the fishery industry. We believe our IFPM can significantly assist in the efficiency of the fishery's planning process.

Apart from generating optimal solutions for a given set of data, the model is extremely useful in answering what-if questions posed by the decision maker. It can be used to evaluate the impact of trawlers and plant capacity variations, quota level variations, to name a few, on potential profit of the fishery. Currently we are unaware of any application in the industry of an effective scheme for performing these evaluations. Hence the availability of the fishery model as a tool to assist in answering these important questions is another contribution made by this thesis.

We also analysed the workability of a rolling horizon approach to solve the longer planning horizon models and to deal with the end-of-planning horizon effects. We found that the rolling horizon was not a good way to decompose the IFPM. Numerical results for several planning horizon models with four different problem instances were presented.

This thesis found ways to manage variability through inventory control. To deal with the end-of-planning-horizon effects, we use a safety stock approach. We investigated the effect of this approach on the profit of an integrated fishery and discovered that the safety stock approach is effective to reduce the end-effects-of -planning horizon.

The fishery can use the IFPM to examine the quality of its recent decisions, say over the last three or four months, using actual catch rates, trawler scheduling, and the best production plans can be produced. The discrepancies between the model's decisions and the company's corresponding decisions can be used to understand, control, and improve the quality of the decision making.

We also investigated the effect of initial and final position of the trawlers on the profit in Chapter 4. From the solution of different planning horizon models, we found that, unlike with the previous assumption, with the repeated (cycling) trawler schedule, the fishery needs a much lower inventory of raw materials. We also observed that, the longer the planning horizon, the smaller the differences in the objective function values.

Another significant contribution of the IFPM which can be easily overlooked is its implications for data collection procedure within the fishery. In testing the IFPM, we faced a major problem of finding the right data sets in the right form. The IFPM specifies what data should be collected and how it should be collected by the fishery. This is an important contribution towards improving and maintaining effective planning procedures within the fishery.

The contributions made in this thesis are not in the form of extensions to existing models for the planning issues, but rather in the form of a new model. Hence the work embodied in this thesis is new.

The implementations of the entire framework were demonstrated in a deterministic setting. The issue that can be created by real-time implementation, understandably, could not be demonstrated.

The second objective of this thesis was to develop efficient solution procedures for the IFPM developed in Chapter 3. The IFPM is hard to solve directly. The IFPM was decomposed into a trawler scheduling subproblem, a processing subproblem, and complicating side constraints. To solve the IFPM, we tried LP relaxation, Lagrangean relaxation (LR), subgradient optimization (SO), Dantzig-Wolfe decomposition (DWD) and decomposition-based pricing (DBP) in Chapter 5. The purpose was to gain insight into the effectiveness of these methods as a mechanism to solve the

IFPM. We developed a new DBP procedure. We observed that the DBP procedure for the IFPM is the most effective method taken in to consideration in Chapter 5. It gives good computation times.

To improve the DBP solution from Chapter 5, we developed a decomposition-based O'Neill pricing (DBONP) procedure in Chapter 6. It improved the optimal profit, but took longer time and more iterations to solve the IFPM.

We finally developed a reduced cost-based pricing (RCBP) algorithm in Chapter 7. This algorithm proved to be quicker to solve our problem and produced better results than all other methods proposed in this thesis. We demonstrated the procedure on several planning horizon models of different problem instances. The RCBP approach bears similarities to other work, e.g., simplex method, DWD, and DBP, but those are all for linear programs, while our RCBP is intended for IPs. This new algorithm extends results from the O'Neill et al (2005) paper, which is about pricing an optimal solution for a market with non-convexities.

Limitations and future work

The limitations of the model lie in the fact that not all pertinent issues can be addressed within the framework of static model. There are several decisions that are hard to quantify. For example, the assignment of captains of trawlers to catching task requires a model that incorporates the skill of a captain. Also how should the trawler schedule be altered in the wake of poor catch rates on fishing grounds?
Another limitation of the model is that they ignore the weather conditions. The observation of weather conditions, like catch rates, is associated with probability

distribution. We choose to ignore its effects because the trawlers generally can fish under most weather scenarios.

Another limitation of the model is that it ignores by-catch. We choose to ignore the by-catch because if by-catch of a fish species is used up for a given area, the fishery can fish in that area by leasing or swapping the quota of that fish species in that area. The fishery can also pay a “deemed value”, basically a fine to fish in that area.

In-depth case studies of NZ fishery companies, comparing the use of the IFPM with their existing practice would be fruitful. The issues created by real-time implementation of the IFPM ought to be investigated. Issues such as the incorporation of new data into the model as information on catch rates are updated.

More challenging problem instances need to be investigated in order to further distinguish the behaviour of the three solution techniques (DBP, DBONP, and RCBP) applied to solving such instances, and also to distinguish which data are more challenging and which data are less challenging. A classification of problem instances “by type” would prove the greatest challenge.

The three solution procedures developed in this thesis, can be applied to other IPs. Our focus was the fishery, so other types of models were outside the scope of this thesis, but they could be interesting future work.

Making the solution procedure interactive through well-designed user interfaces would be extremely fruitful.

Concluding remarks

The intent of this thesis is not to replace the decision maker with a mathematical model, given that they are imperfections of the real world, but rather to provide structured and reliable support for the decision maker who ultimately makes the final decision. The current state of this $2 billion industry in NZ is one in which no support system of this kind is in use.

We therefore believe that this thesis is a contribution towards the goal of a quota-based integrated commercial fishery in New Zealand, and in fact, anywhere in the world where the fisheries are running under quota allocations. Our work would also apply just as well if a fishery with the same in all respects, but did not have the quota constraint. We hope that this thesis will add value to Operations Research and to the fishery industry, and will fill an important knowledge gap.

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Appendix 1

Description of an AMPL Model

A1. 1 Introduction

In this appendix, we present a sample model implementation in the AMPL (Fourer et al, 1993) modelling language and CPLEX. AMPL stands for “A Modelling Language for Mathematical Programming”. AMPL allows for separation of the data from the model by inputting each as a separate file. CPLEX is an optimization package for LP, IP, or MILP. It uses the simplex algorithm for LP and a branch-and-bound approach for MILP.

The AMPL algorithm model and run files for the methods developed in this thesis can be available on request. The email addresses are: b.hasan@mang.canterbury.ac.nz and mbabulhasan@yahoo.com .

The remainder of this appendix is organized as follows. Section A1.2 presents the formulation of the model file. In section A1.3, we present a sample data file for a 10-period model. In section A1.4, we present a run file. And in section A5, we present an out put file from a 10-period model.

A1.2 Model file formulation

In this section, we present the model file consisting of its indexed sets, parameters, decision variables, objective function and constraints.

A1.2.1 Indexed sets

SETS	index	
set raw;	i	# set of raw materials.
set products;	j	# set of products.
set trawler;	v	# set of vessels.
set periods :=1..T;	t, u, s	# set of periods.
set centres;	c	# set of work centres.
set quality;	q	# set of quality of landed fish
set stocks;	a	# set of fishing locations
set factory;	p	# set of factory

set trips:= { stocks, periods, 1..T, trawler: t > u and t-u <= N[v]};

A1.2.2 Parameters for fishing

param A {trawler}>= 0;	# capacity of trawler v
param BIF {raw, quality};	# beginning inventory fish
param C {raw} >= 0;	# cost of landed fish
param E {stocks, raw, periods, trawler};	# average catch per day

param ET{a in stocks, u in 1..T, t in 1..T, v in trawler: t>u and t-u <= N[v]}:=

max (0, min(A[v], sum {i in raw, s in periods: s >= u and s <= t-1 and (a, u, t, v) in trips} E[a, i, s, v]- sum{i in raw}TR[a, v]*E[a, i, u, v]- sum {i in raw} TR[a, v]*E[a,i,t-1,v]));

param I >=0 := 0.025; # inventory cost in each period

param MI >= 0 := 150000; # maximum kilograms of inventory raw materials

param V {trawler}; # vessel operating cost

param FRaw {stocks, raw, trawler}; # fraction of different each species

param T>0 integer; # number of periods in a planning horizon.

param N {trawler} >= 0; # maximum no. of fishing days in each period

param TR {stocks, trawler} >= 0 ; # time required for trip

A1.2.3 Parameters for processing

param BIP {raw, products, quality}; # beginning inventory product.

param F {raw, products} >= 0; # kg raw material needed to produce 1 kg product

param FR {raw, quality} >= 0; # fraction of different quality of each species

param H {raw, products} >=0; # labour hour

param J >= 0 := 0.025; #inventory holding cost for product

param Lr{periods} >= 0; #regular labour cost for each period

param Lo >= 0 := 25; # overtime labour cost for each period

param LAr >=0; # lower bound on available regular labour hour

param LAo {periods}>=0; # lower bound on available overtime labour hour

param LM {raw, products, quality, periods} >= 0; # lower bounds on product param LR {raw, quality, periods} >= 0; # lower bounds on raw material to be processed

param MIP >= 0 := 150000; # maximum inventory storage capacity

param P {raw, products, quality} >=0 ; # profit of processing j for i

param UAr>=0; # upper bounds on regular labour in period t

param UAo {periods}>= 0;

param UM {raw, products, quality, periods} >= 0; # upper bounds on product

param UR {raw, quality, periods} >=0; # upper bounds on raw material

param LS{raw, products, quality} >= 0; # lower bounds on sell

param US{raw, products, quality} >= 0; # upper bound

A1.2.4 Parameters for quota

param G {stocks, aw}; # quota left from earlier trading and fishing

A1.2.5 Variables for fishing

var w {p in factory, a in stocks, u in periods, t in 1..T,v in trawler: t>u and t-u <=N[v] and (a,u,t,v) in trips} binary; # number of days in period t

var wr {t in periods, v in trawler} binary;

var z {raw, quality, 0..T} >= 0; # inventory landed fish in period t

var f {stocks, raw, quality, periods, trawler} >= 0; # kg of fish species i landed.

A1.2.7 Variables for processing

var r {raw,products,quality,0..T} >= 0; # inventory products

var s {raw, products, quality, periods} >= 0; # product sold

var x {raw, products, quality, periods} >= 0; # kg of product j produced in period t

var yr >= 0; #regular labour hours in period t var yo {periods} >= 0; # overtime labour hours in period t

A1.2.8 Variables for quota

var q {stocks,raw,0..T} >= 0; # inventory quota for next periods

A1.2.9 Objective function

maximize total_profit:

- sum {p in factory} sum { a in stocks, u in 1..T, t in 1..T, v in trawler: (a, u, t, v) in trips} (t-u)*w[p, a, u, t,v]* V[v] - sum {a in stocks} sum {i in raw} sum {t in 1..T} sum {l in quality} sum {v in trawler} C[i]*f[a, i, l, t, v] + sum {i in raw} sum {j in products} sum {l in quality} sum {t in 1..T} P[i, j, l] * s[i, j, l, t]- sum {t in 1..T} Lr[t]* yr - sum {t in 1..T} Lo * yo[t] - sum {i in raw} sum {l in quality} sum {t in 1..T} I * z[i, l, t]- sum {i in raw} sum {j in products} sum {l in quality} sum {t in 1..T} J * r[i, j, l, t];

A1.2.10 Constraints

subject to landed_fish {p in factory, a in stocks, i in raw, l in quality, t in 1..T,v in trawler}:

f[a,i,l,t,v] = sum {u in 1..T: t>u and t-u<=N[v] and (a, u, t, v)in trips} ET[a, u, t, v] *w[p, a, u, t, v]*FR[i,l]* FRaw[a, i, v];

subject to trawl_start {p in factory, v in trawler}:

sum {a in stocks} sum {t in 1..T: (a,1,t,v) in trips} w[p,a,1,t,v] + wr[1,v]= 1;

subject to flow {p in factory, t in 2..T, v in trawler}:

```
sum{a in stocks} sum {u in 1..T: (a, u, t, v) in trips} w[p, a, u, t, v] + wr[t-1,v]
- sum {a in stocks} sum {t1 in 1..T: (a,t,t1,v) in trips} w[p,a,t,t1,v]- wr[t, v] = 0;

subject to inventory_balance {i in raw, l in quality, t in 1..T}:

    z[i,l,t-1] + sum {a in stocks} sum {v in trawler} f[a, i, l, t, v]-

    sum {j in products} F[i, j]* x[i, j, l, t] =  z[i, l, t];

                        # unused fish is stored as inventory for next period

subject to store {t in 1..T}:

    sum {i in raw} sum {l in quality} z[i, l, t] <= MI;    # limited storage for
                                                            inventory

subject to end_effect {i in raw, l in quality, t in 1..T}:     z[i,l,0] =  z[i, l, T];

subject to product_sell {i in raw, j in products, l in quality, t in 1..T}:

    s[i, j, l, t] <= US[i, j, l];

subject to initial_inventory_product {i in raw, j in products, l in quality}:   r[i,j,l,0] =
BIP[i, j, l];

subject to inventory_product {i in raw, j in products, l in quality, t in 1..T}:

    r[i,j,l,t-1] + x[i, j, l, t]- s[i, j, l, t] = r[i, j, l, t];

                        # unused product is stored as inventory for next period

subject to product_store {i in raw, t in 1..T}:

    sum {j in products} sum {l in quality} r[i, j, l, t] <= MIP;

                        # limited storage for inventory products
```

subject to work_time {t in 1..T}:

sum {i in raw} sum {j in products} sum {l in quality} H[i, j] * x[i, j, l, t] - yr-yo[t] <= 0;

work time in fleeting packing etc.

subject to uper_bound {t in periods}:

yo[t] <= 0.25* yr;

subject to initial_quota {a in stocks, i in raw}:

q[a,i,0] = G[a, i];

subject to quota_balance {a in stocks, i in raw, t in 1..T }:

q[a,i,t-1] - sum {l in quality} sum {v in trawler} f[a, i, l, t, v] = q[a, i, t] ;

A1.3 Data file formulation

In this section, we present a sample data file for a 10-period model.

```
set raw :=          hoki   roughy   dory   ling   redcod   squid   bcuda
efish;
set products :=     FILL   GUT   HGU;
set trawler :=      tr1    tr2   tr3;
set quality :=      type1  type2 type3;
set stocks :=       area1  area2  area3  area4  area5  area6  area7  area8
        area9  area10;
set factory :=      timaru;
```

 # Capacity if trawler v

```
param A :=
        tr1     85000
        tr2     35000
        tr3     35000;
```

 # Average expected catch

```
param E default 0:=

[area3, *, *, tr1]

:   bcuda   dory   efish  ling  redcod roughy squid :=

1   15000   25000  1000   500   5000   15000  10000

2   15000   25000  1000   500   5000   15000  10000

3   15000   25000  1000   500   5000   15000  10000

4   15000   25000  1000   500   5000   15000  10000

5   15000   25000  1000   500   5000   15000  10000

6   15000   25000  1000   500   5000   15000  10000

7   15000   25000  1000   500   5000   15000  10000

8   15000   25000  1000   500   5000   15000  10000

9   15000   25000  1000   500   5000   15000  10000

10  15000   25000  1000   500   5000   15000  10000

[area3,*,*,tr2]

:   bcuda   dory   efish  ling  redcod roughy squid :=

1   15000   15000  1000   100   5000   10000  10000

2   15000   15000  1000   100   5000   10000  10000

3   15000   15000  1000   100   5000   10000  10000

4   15000   15000  1000   100   5000   10000  10000

5   15000   15000  1000   100   5000   10000  10000

6   15000   15000  1000   100   5000   10000  10000

7   15000   15000  1000   100   5000   10000  10000

8   15000   15000  1000   100   5000   10000  10000
```

9	15000	15000	1000	100	5000	10000	10000
10	15000	15000	1000	100	5000	10000	10000

[area3,*,*, tr3]

:	bcuda	dory	efish	ling	redcod	roughy	squid :=
1	15000	15000	1000	100	5000	10000	10000
2	15000	15000	1000	100	5000	10000	10000
3	15000	15000	1000	100	5000	10000	10000
4	15000	15000	1000	100	5000	10000	10000
5	15000	15000	1000	100	5000	10000	10000
6	15000	15000	1000	100	5000	10000	10000
7	15000	15000	1000	100	5000	10000	10000
8	15000	15000	1000	100	5000	10000	10000
9	15000	15000	1000	100	5000	10000	10000
10	15000	15000	1000	100	5000	10000	10000;

[area7,*,*,tr1]			[area7,*,*,tr2]			[area7, *, *, tr3]		
:	hoki	ling:=	:	hoki	ling:=	:	hoki	ling:=
1	40000	1000	1	20000	500	1	20000	500
2	40000	1000	2	20000	500	2	20000	500
3	40000	1000	3	20000	500	3	20000	500
4	40000	1000	4	20000	500	4	20000	500
5	40000	1000	5	20000	500	5	20000	500
6	40000	1000	6	20000	500	6	20000	500
7	40000	1000	7	20000	500	7	20000	500
8	40000	1000	8	20000	500	8	20000	500
9	40000	1000	9	20000	500	9	20000	500
10	40000	1000	10	20000	500	10	20000	500

 # Fraction of each species landed

param FRaw :=

[area3,*,*]:	tr1	tr2	tr3:=
hoki	0	0	0
roughy	0.210	0.1785	0.1785
dory	0.350	0.267	0.267
ling	0.007	0.002	0.002
redcod	0.070	0.089	0.089
squid	0.140	0.1785	0.1785
bcuda	0.210	0.267	0.267
efish	0.013	0.018	0.018

[area7,*,*]:	tr1	tr2	tr3:=
hoki	.975	0.976	0.976
roughy	0	0	0
dory	0	0	0
ling	0.025	0.024	0.024
redcod	0	0	0
squid	0	0	0
bcuda	0	0	0
efish	0	0	0;

 # N = maximum no. of fishing days in each period

param N :=

tr1	60
tr2	21
tr3	21;

param TR default 0:

	tr1	tr2	tr3:=
area3	0.5	0.5	0.5
area7	0.75	0.75	0.75;

 # V = vessel operating cost

param V :=

tr1	6000
tr2	3500
tr3	3500;

 # F = kg of raw material needed to produce 1 kg product

param F :

	FILL	GUT	HGU:=
hoki	2.65	1.1	1.5
roughy	3.5	1.1	2.0
dory	2.6	1.1	1.5
ling	2.8	1.15	1.45
redcod	2.5	1.1	1.65
squid	0	1.35	1.9
bcuda	2.3	1.1	1.45
efish	2.85	1.1	2.3;

```
param FR:      type1   type2   type3:=

        hoki   .33     .33     .33

        roughy .33     .33     .33

        dory   .33     .33     .33

        ling   .33     .33     .33

        redcod .33     .33     .33

        squid  .33     .33     .33

        bcuda  .33     .33     .33

        efish  .33     .33     .33;

param H default 0:     FILL    GUT     HGU:=

               hoki    0.041   0.032   0.032

               roughy  0.09    0.07    0.073

               dory    0.09    0.07    0.073

               ling    0.05    0.04    0.041

               redcod  0.09    0.07    0.073

               squid   0.0     0.02    0.026

               bcuda   0.03    0.02    0.02

               efish   0.09    0.07    0.073;
```

 # P = Selling price of product j from raw material I and quality l.

```
param P default 0:=

        [hoki,*,*]:    type1   type2   type3:=

               FILL    5.5     4.5     2.5

               GUT     1.0     1.0     0.9

               HGU     3.2     2.5     0.9
```

```
[roughy,*,*]:  type1  type2  type3:=
               FILL   18.0   16.5   12.5
               GUT    4.0    3.5    0.05
               HGU    7.0    6.0    0.05
[dory,*,*]:    type1  type2  type3:=
               FILL   7.0    6.0    4.0
               GUT    1.5    1.3    0.05
               HGU    2.8    2.5    0.05
[ling,*,*]:    type1  type2  type3:=
               FILL   10.0   8.5    6.0
               GUT    2.5    2.3    0.05
               HGU    4.0    3.5    0.05
[redcod,*,*]:  type1  type2  type3:=
               FILL   5.5    4.5    2.5
               GUT    1.0    1.0    0.9
               HGU    3.2    2.5    0.9
[squid,*,*]:   type1  type2  type3:=
               FILL   0      0      0
               GUT    4.0    3.5    0.05
               HGU    7.0    6.0    0.05
[bcuda,*,*]:   type1  type2  type3:=
               FILL   7.0    6.0    4.0
               GUT    1.5    1.3    0.05
               HGU    2.8    2.5    0.05
```

```
[efish,*,*]:    type1   type2   type3:=
                FILL    10.0    8.5     6.0
                GUT     2.5     2.3     0.05
                HGU     4.0     3.5     0.05;
```

 # G = quota left from earlier trading and fishing

```
param G default 0:
         hoki      roughy  dory    ling    redcod   squid     bcuda    efish:=
 area3   0         150000  400000  20000   3500000  25000000  4000000  40000
 area7   27000000  10000   10000   50000   10000    30000     700000   5000;
```

A1.4 Run file

This section presents the run file of the sample model.

```
option show_stats 1;
model FM1mod.txt;    data FM1dat.txt;
printf "\n\nStartTime %s.\n", ctime();
option omit_zero_rows 1, omit_zero_cols 1;
display _nvars; display _ncons;
solve;
printf "Period\tBegininv\tArrived\t\tWorked\t\tEnd inv\t work hour\n"> ma.txt;
for {t in 1..T}
{       printf "%i,\t", t> ma.txt;
        printf "%7.0f,\t", sum {i in raw, l in quality} z[i, l, t-1]> ma.txt;
        printf "%7.0f,\t", sum {a in stocks, i in raw, l in quality, v
                in trawler} f[a, i, l, t, v]> ma.txt;
        printf "%7.0f,\t", sum {i in raw, j in products, l in quality}
                F[i,j]* x[i, j, l, t]> ma.txt;
        printf "%7.0f\t", sum {i in raw, l in quality} z[i, l, t]> ma.txt;
        printf "%7.0f\n", sum {i in raw, j in products, l in quality}
                H[i, j]*x[i, j, l, t]> ma.txt;
}
printf "Period\tBIP \tProduce\tSell\tEndIP\tRhused\tOvertime\n" > ma.txt;
for {t in 1..T}
{       printf "%i,\t", t> ma.txt;
        printf "%5.0f,\t",sum {i in raw, j in products, l in quality} r[i, j, l, t-1]> ma.txt;
```

```
printf "%5.0f\t", sum {i in raw, j in products, l in quality} x[i, j, l, t]> ma.txt;
printf "%5.0f\t", sum {i in raw, j in products, l in quality} s[i, j, l, t]> ma.txt;
printf "%5.0f,\t",sum {i in raw, j in products, l in quality} r[i, j, l, t]> ma.txt;
printf "%7.0f\t", sum {i in raw, j in products, l in quality} H[i, j]*x[i, j, l, t] -
yo[t]> ma.txt;
printf "%5.0f,\n", yo[t]> ma.txt;
}
display yr;
display sum {t in periods} yo[t];
display sum {i in raw, l in quality, t in 1..T} z[i, l, t];
display sum {a in stocks, i in raw, l in quality, t in periods, v in trawler} f[a, i, l, t, v];
printf "Period\tregular\tover\traw\tprod\trevenue\ttrawler\n"; for {t in 1..T}

{       printf "%i,\t", t;
        printf "%5.0f,\t", Lr[t]*yr;
        printf "%5.0f,\t", Lo*yo[t];
        printf "%5.0f,\t", sum {i in raw, l in quality} I*z[i, l, t];
        printf "%5.0f\t", sum {i in raw, j in products, l in quality} J*r[i,j, l, t];
        printf "%5.0f\t", sum {i in raw, j in products, l in quality} P[i, j, l]*s[i, j, l, t];
        printf "%5.0f\n", sum {p in factory, a in stocks, u in 1..T, v in trawler: (a, u, t,
v) in trips}
 (t-u)*w[p, a, u, t, v]*V[v];
}
display sum {t in periods} Lr[t]*yr; display sum{t in periods} Lo*yo[t];
display sum {i in raw, l in quality, t in periods} I*z[i, l, t];
```

```
display sum {i in raw, j in products, l in quality, t in periods} J*r[i, j, l, t];

display sum {i in raw, j in products, l in quality, t in periods} P[i, j, l]*s[i, j, l, t];

display sum {p in factory, a in stocks, u in 1..T,t in periods, v in trawler: (a, u, t, v) in
trips}
 (t-u)*w[p, a, u, t, v]*V[v];

printf "Period\tArrived\n";

for {t in 1..T, v in trawler}
        {       printf "%i,\t", t> ma.txt;
                printf "%7.0f,\n", sum {a in stocks, i in raw, l in quality} f[a, i, l, t, v]>
ma.txt;
        }

printf "CompletionTime %s.\n", ctime();
```

A1.5 Output

total_profit = $1,065,775

w :=

timaru area3 1 3 tr2 1

timaru area3 1 3 tr3 1

timaru area3 1 4 tr1 1

timaru area3 3 5 tr2 1

timaru area3 4 6 tr1 1

timaru area3 4 6 tr3 1

timaru area3 6 8 tr1 1

timaru area3 6 8 tr2 1

timaru area3 6 8 tr3 1

timaru area3 8 10 tr1 1

timaru area3 8 10 tr2 1

timaru area3 8 10 tr3 1;

f [area3,*,type1,*,tr1]

:	bcuda	dory	efish	ling	redcod	roughy	squid	:=
4	5890.5	9817.5	364.65	196.35	1963.5	5890.5	3927	
6	4954.95	8258.25	306.735	165.165	1651.65	4954.95	3303.3	
8	4954.95	8258.25	306.735	165.165	1651.65	4954.95	3303.3	
10	4954.95	8258.25	306.735	165.165	1651.65	4954.95	3303.3	

```
[area3,*,type1,*,tr2]
:   bcuda    dory    efish  ling  redcod   roughy    squid    :=
3   3083.85   3083.85   207.9   23.1   1027.95   2061.68   2061.68
5   3083.85   3083.85   207.9   23.1   1027.95   2061.68   2061.68
8   3083.85   3083.85   207.9   23.1   1027.95   2061.68   2061.68
10   3083.85   3083.85   207.9   23.1   1027.95   2061.68   2061.68
[area3,*,type1,*,tr3]
:   bcuda    dory    efish  ling  redcod   roughy    squid    :=
3   3083.85   3083.85   207.9   23.1   1027.95   2061.67   2061.67
6   3083.85   3083.85   207.9   23.1   1027.95   2061.68   2061.68
8   3083.85   3083.85   207.9   23.1   1027.95   2061.68   2061.68
10   3083.85   3083.85   207.9   23.1   1027.95   2061.68   2061.68
[area3,*,type2,*,tr1]
:   bcuda    dory    efish    ling    redcod   roughy   squid    :=
4   5890.5    9817.5    364.65    196.35    1963.5    5890.5    3927
6   4954.95   8258.25   306.735   165.165   1651.65   4954.95   3303.3
8   4954.95   8258.25   306.735   165.165   1651.65   4954.95   3303.3
10   4954.95   8258.25   306.735   165.165   1651.65   4954.95   3303.3
[area3,*, type2,*,tr2]
:   bcuda    dory    efish  ling  redcod   roughy    squid    :=
3   3083.85   3083.85   207.9   23.1   1027.95   2061.68   2061.68
5   3083.85   3083.85   207.9   23.1   1027.95   2061.68   2061.68
8   3083.85   3083.85   207.9   23.1   1027.95   2061.68   2061.68
10   3083.85   3083.85   207.9   23.1   1027.95   2061.68   2061.68
```

[area3,*, type2,*,tr3]

: bcuda dory efish ling redcod roughy squid :=

3 3083.85 3083.85 207.9 23.1 1027.95 2061.67 2061.67

6 3083.85 3083.85 207.9 23.1 1027.95 2061.68 2061.68

8 3083.85 3083.85 207.9 23.1 1027.95 2061.68 2061.68

10 3083.85 3083.85 207.9 23.1 1027.95 2061.68 2061.68

[area3,*,type3,*,tr1]

: bcuda dory efish ling redcod roughy squid :=

4 5890.5 9817.5 364.65 196.35 1963.5 5890.5 3927

6 4954.95 8258.25 306.735 165.165 1651.65 4954.95 3303.3

8 4954.95 8258.25 306.735 165.165 1651.65 4954.95 3303.3

10 4954.95 8258.25 306.735 165.165 1651.65 4954.95 3303.3

[area3,*, type3,*,tr2]

: bcuda dory efish ling redcod roughy squid :=

3 3083.85 3083.85 207.9 23.1 1027.95 2061.68 2061.68

5 3083.85 3083.85 207.9 23.1 1027.95 2061.68 2061.68

8 3083.85 3083.85 207.9 23.1 1027.95 2061.68 2061.68

10 3083.85 3083.85 207.9 23.1 1027.95 2061.68 2061.68

[area3,*,type3,*,tr3]

: bcuda dory efish ling redcod roughy squid :=

3 3083.85 3083.85 207.9 23.1 1027.95 2061.67 2061.67

6 3083.85 3083.85 207.9 23.1 1027.95 2061.68 2061.68

8 3083.85 3083.85 207.9 23.1 1027.95 2061.68 2061.68

10 3083.85 3083.85 207.9 23.1 1027.95 2061.68 2061.68;

z [*,type1,*]

:	bcuda	dory	efish	redcod	roughy	squid	:=
0	8626.15	16670.1	722.535	2574.69	3828.3	5700	
1	0	10520.1	722.535	2500	0	2850	
3	0	2267.7	0	0	0	0	
4	0	8055.33	0	293.957	0	0	
5	0	1089.18	0	0	0	0	
6	0	10181.3	0	2092.84	1766.62	0	
8	0	14412.9	0	1207.55	3828.3	0	
9	0	8394.18	0	0	0	0	
10	8626.15	16670.1	722.535	2574.69	3828.3	5700	

[*,type2,*]

:	bcuda	dory	efish	redcod	roughy	squid	:=
0	8626.15	16538.9	722.535	3707.55	3828.3	5700	
1	0	10388.9	722.535	2500	0	2850	
3	0	603.792	0	0	0	0	
4	0	7924.05	0	0	0	0	
5	0	3166.93	0	0	0	0	
6	0	10050	0	2500	1766.62	0	
8	0	14412.9	0	2329.62	3828.3	0	
9	0	8262.9	0	0	0	0	
10	8626.15	16538.9	722.535	3707.55	3828.3	5700	

[*,type3,*]

:	bcuda	dory	efish	redcod	roughy	squid :=
0	8626.15	7800	722.535	3707.55	3828.3	2551.65
1	0	3900	722.535	2500	0	0
3	0	2267.7	0	0	0	0
4	0	8185.2	0	0	0	0
5	0	3469.05	0	0	0	0
6	0	14811.1	0	179.6	0	0
7	0	2296.09	0	0	0	0
8	0	16722	0	1207.55	3828.3	0
9	0	2022.52	0	0	0	0
10	8626.15	7800	722.535	3707.55	3828.3	2551.65;

x [*,FILL,type1,*]

:	bcuda	dory	efish	ling	redcod	roughy
:=						
1	3750.5	1500	0	0	29.8778	1093.8
2	0	1500	253.521	0	1000	0
3	2681.61	1500	145.895	16.5	822.36	1178.1
4	2561.09	1500	127.947	70.125	667.817	1683
5	1340.8	3000	72.9474	8.25	528.763	589.05
6	3495.13	0	180.574	67.2375	234.703	1500
7	0	3050.49	0	0	837.137	504.75
8	4835.93	0	253.521	75.4875	1000	1500
9	0	1449.51	0	0	483.02	1093.8
10	1085.43	1500	0	75.4875	453.142	1500

[*,FILL,type2,*]

:	bcuda	dory	efish	ling	redcod	roughy
:=						
1	3750.5	1500	0	0	483.02	1093.8
2	0	1500	253.521	0	1000	0
3	2681.61	2139.96	145.895	16.5	822.36	1178.1
4	2561.09	860.035	127.947	70.125	785.4	1683
5	1340.8	2150.37	72.9474	8.25	411.18	589.05
6	3495.13	849.628	180.574	67.2375	71.84	1500
7	0	3000	0	0	1000	504.75
8	4835.93	0	253.521	75.4875	551.173	1500
9	0	1500	0	0	931.847	1093.8
10	1085.43	1500	0	75.4875	0	1500

[*,FILL,type3,*]

:	bcuda	dory	efish	ling	redcod	roughy :=
1	3750.5	1500	0	0	483.02	1093.8
2	0	1500	253.521	0	1000	0
3	2681.61	1500	145.895	16.5	822.36	1178.1
4	2561.09	1500	127.947	70.125	785.4	1683
5	1340.8	3000	72.9474	8.25	411.18	589.05
6	3495.13	0	180.574	67.2375	1000	2004.75
7	0	4813.49	0	0	71.84	0
8	4835.93	0	253.521	75.4875	1000	1500
9	0	5653.66	0	0	483.02	1093.8
10	1085.43	3326.34	0	75.4875	0	1500

```
 [*,GUT,type1,*]
:    squid    :=
3    908.889
8    446.889
 [*,GUT,type2,*]
:    squid    :=
3    908.889
8    446.889
[*,GUT, type3,*]
:    squid     :=
3     943.222
4     213.833
6    1500
8    1500
10   1500
 [*,HGU,type1,*]
:     dory        squid      :=
1    1500         1500
2    4413.42      1500
3       0         1524.39
4      86.5817    2066.84
5    1500         1085.09
6    1500         2823.67
7    1500          0
8       8.7       3591.24
```

```
9    1500         0
10   1500       908.763
 [*,HGU, type2,*]
:    dory         squid     :=
1    1500         1500
2    4325.9       1500
3      0          1524.39
4     174.1       2066.84
5    1500         1085.09
6    1500         2823.67
7    1500         0
8      8.7        3591.24
9    1500        0
10   1500       908.763
 [*,HGU,type3,*]
:    squid    :=
1    1342.97
3    1500
4    1914.91
5    1085.09
6    1757.88
8    2842.97
10   1500;
```

```
r [*,FILL,type1,*]
:    bcuda      dory           roughy    :=
1    1750.5      0              0
3    681.609     0              0
4    1242.7      0              183
5    583.5       1500           0
6    2078.63     0              0
7    78.6304     1550.49        0
8    2914.57     50.4913        0
9    914.565     0              0
 [*,FILL,type2,*]
:    bcuda      dory           roughy    :=
1    1750.5      0              0
3    681.609     639.965        0
4    1242.7      0              183
5    583.5       650.372        0
6    2078.63     0              0
7    78.6304     1500           0
8    2914.57     0              0
9    914.565     0              0
[*,FILL,type3,*]
:    bcuda      dory           roughy    :=
1    1750.5      0              0
3    681.609     0              0
4    1242.7      0              183
```

5	583.5	1500	0
6	2078.63	0	504.75
7	78.6304	3313.49	0
8	2914.57	1813.49	0
9	914.565	5967.15	0
10	0	7793.48	0

[*,HGU,type1,*]

:	dory	squid	:=
2	2913.42	0	
3	1413.42	24.3947	
4	0	591.237	
5	0	176.329	
6	0	1500	
8	0	2091.24	
9	0	591.237	

[*,HGU, type2,*]

:	dory	squid	:=
2	2825.9	0	
3	1325.9	24.3947	
4	0	591.237	
5	0	176.329	
6	0	1500	
8	0	2091.24	
9	0	591.237	

```
[*,HGU, type3,*]
:   squid   :=
4   414.908
6   257.882
8  1342.97;
s [*,FILL,type1,*]
:  bcuda   dory   efish   ling   redcod   roughy   :=
1   2000   1500   0   0   29.8778   1093.8
2   1750.5   1500   253.521   0   1000   0
3   2000   1500   145.895   16.5   822.36   1178.1
4   2000   1500   127.947   70.125   667.817   1500
5   2000   1500   72.9474   8.25   528.763   772.05
6   2000   1500   180.574   67.2375   234.703   1500
7   2000   1500   0   0   837.137   504.75
8   2000   1500   253.521   75.4875   1000   1500
9   2000   1500   0   0   483.02   1093.8
10   2000   1500   0   75.4875   453.142   1500
 [*,FILL,type2,*]
:  bcuda   dory   efish   ling   redcod   roughy   :=
1   2000   1500   0   0   483.02   1093.8
2   1750.5   1500   253.521   0   1000   0
3   2000   1500   145.895   16.5   822.36   1178.1
4   2000   1500   127.947   70.125   785.4   1500
5   2000   1500   72.9474   8.25   411.18   772.05
6   2000   1500   180.574   67.2375   71.84   1500
```

7	2000	1500	0	0	1000	504.75
8	2000	1500	253.521	75.4875	551.173	1500
9	2000	1500	0	0	931.847	1093.8
10	2000	1500	0	75.4875	0	1500

[*,FILL,type3,*]

:	bcuda	dory	efish	ling	redcod	roughy :=
1	2000	1500	0	0	483.02	1093.8
2	1750.5	1500	253.521	0	1000	0
3	2000	1500	145.895	16.5	822.36	1178.1
4	2000	1500	127.947	70.125	785.4	1500
5	2000	1500	72.9474	8.25	411.18	772.05
6	2000	1500	180.574	67.2375	1000	1500
7	2000	1500	0	0	71.84	504.75
8	2000	1500	253.521	75.4875	1000	1500
9	2000	1500	0	0	483.02	1093.8
10	2000	1500	0	75.4875	0	1500

[*, GUT, type1,*]

:	squid :=
3	908.889
8	446.889

[*, GUT, type2,*]

:	squid :=
3	908.889
8	446.889

[*,GUT,type3,*]

: squid :=

3 943.222

4 213.833

6 1500

8 1500

10 1500

[*, HGU, type1,*]

: dory squid :=

1 1500 1500

2 1500 1500

3 1500 1500

4 1500 1500

5 1500 1500

6 1500 1500

7 1500 1500

8 8.7 1500

9 1500 1500

10 1500 1500

[*, HGU, type2,*]

: dory squid :=

1 1500 1500

2 1500 1500

3 1500 1500

4 1500 1500

```
5    1500     1500
6    1500     1500
7    1500     1500
8      8.7   1500
9    1500     1500
10   1500     1500
 [*, HGU, type3,*]
:    squid      :=
1    1342.97
3    1500
4    1500
5    1500
6    1500
7     257.882
8    1500
9    1342.97
10   1500;
yr = 1459.42;
yo [*] := 0;
```

q [area3,*,*]:

	bcuda	dory	efish	ling	redcod	roughy	squid :=
0	4e+06	4e+05	40000	20000	3500000	150000	2.5e+07
1	4e+06	4e+05	40000	20000	3500000	150000	2.5e+07
2	4e+06	4e+05	40000	20000	3500000	150000	2.5e+07
3	3981500	381497	38752.6	19861.4	3493830	137630	24987600
4	3963830	352044	37658.6	19272.4	3487940	119958	24975800
5	3954570	342793	37034.9	19203.1	3484860	113773	24969700
6	3930460	308767	35491	18638.3	3476820	92723.6	24953600
7	3930460	308767	35491	18638.3	3476820	92723.6	24953600
8	3897090	265489	33323.4	18004.2	3465700	65488.7	24931300
9	3897090	265489	33323.4	18004.2	3465700	65488.7	24931300
10	3863720	222211	31155.8	17370.1	3454570	38253.8	24909000

[area7,*,*]

:	bcuda	dory	efish	hoki	ling	redcod	roughy	squid :=
0	7e+05	10000	5000	2.7e+07	50000	10000	10000	30000
1	7e+05	10000	5000	2.7e+07	50000	10000	10000	30000
2	7e+05	10000	5000	2.7e+07	50000	10000	10000	30000
3	7e+05	10000	5000	2.7e+07	50000	10000	10000	30000
4	7e+05	10000	5000	2.7e+07	50000	10000	10000	30000
5	7e+05	10000	5000	2.7e+07	50000	10000	10000	30000
6	7e+05	10000	5000	2.7e+07	50000	10000	10000	30000
7	7e+05	10000	5000	2.7e+07	50000	10000	10000	30000
8	7e+05	10000	5000	2.7e+07	50000	10000	10000	30000
9	7e+05	10000	5000	2.7e+07	50000	10000	10000	30000

10 7e+05 10000 5000 2.7e+07 50000 10000 10000 30000;

Period	Beginning inv.	Arrived	Worked	End inv	work hour
1,	104481,	0,	64305,	40177	1459
2,	40177,	0,	40177,	0	1459
3,	0,	69300,	64161,	5139	1459
4,	5139,	84150,	64831,	24459	1459
5,	24459,	34650,	51383,	7725	1459
6,	7725,	105435,	69812,	43348	1459
7,	43348,	0,	41052,	2296	1459
8,	2296,	140085,	81697,	60684	1459
9,	60684,	0,	42004,	18680	1459
10,	18680,	140085,	54283,	104481	1459

Period	BIP	Produce	Sell	EndIP	Rhused	Overtime
1,	0,	27372	22120	5252,	1459	0,
2,	5252,	20000	19512	5739,	1459	0,
3,	5739,	26983	27250	5473,	1459	0,
4,	5473,	25948	25547	5874,	1459	0,
5,	5874,	21790	21911	5754,	1459	0,
6,	5754,	30295	26050	9999,	1459	0,
7,	9999,	16782	20181	6600,	1459	0,
8,	6600,	34983	24356	17227,	1459	0,
9,	17227,	16782	24116	9893,	1459	0,
10,	9893,	22580	24680	7793,	1459	0,

sum{i in raw, l in quality, t in 1 .. T} z[i,l,t] = 306988

sum{a in stocks, i in raw, l in quality, t in periods, v in trawler} f[a, i, l, t ,v] = 573705

Period	regular	over	raw	prod	revenue	trawler
1,	29188,	0,	1004,	131	142424	0
2,	29188,	0,	0,	143	101420	0
3,	29188,	0,	128,	137	163518	14000
4,	29188,	0,	611,	147	171912	18000
5,	29188,	0,	193,	144	130734	7000
6,	29188,	0,	1084,	250	167623	19000
7,	29188,	0,	57,	165	120133	0
8,	29188,	0,	1517,	431	167184	26000
9,	29188,	0,	467,	247	150888	0
10,	29188,	0,	2612,	195	161489	26000

sum{t in periods} Lr[t]*yr = 291884

sum{t in periods} Lo*yo[t] = 0

sum{i in raw, l in quality, t in periods} I*z[i, l, t] = 7674.71

sum{i in raw, j in products, l in quality, t in periods} J*r[i, , l, t] = 1990.1

sum{i in raw, j in products, l in quality, t in periods} P[i, j, l]*s[i, j, l, t] = 1477320

sum{p in factory, a in stocks, u in 1 .. T, t in periods, v in trawler: (a, u, t ,v) in trips} (t - u)*w[p, a, u, t, v]*V[v] = 110000

Period	Fish landed
3,	34650,
3,	34650,
4,	84150,
5,	34650,
6,	70785,
6,	34650,
8,	70785,
8,	34650,
8,	34650,
10,	70785,
10,	34650,
10,	34650;

Appendix 2

In this appendix, we present one of our papers entitled “A mixed integer linear program for an integrated fishery” published in “South African Journal of Operations Research (*ORiON*)”.

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A mixed integer linear program for an integrated fishery

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Abstract

In this paper fishing trawler scheduling and production planning for a quota-based integrated commercial fishery is modelled mathematically. The catch capacity of fishing trawlers and the capacity of processing firms are two major factors which influence the scheduling of fishing trawlers. Production planning in fish processing firms depends on steady supply of fresh fish from the fishing trawlers to the processing firms. We develop a mixed integer linear programming (MILP) model to co-ordinate trawler scheduling, fishing, processing, and labour allocation of quota based integrated fisheries. We demonstrate the workability of our model with a numerical example and sensitivity analysis based on data obtained from one of the major fisheries in New Zealand.

Key words: Mixed integer linear program, fishing, trawler scheduling, processing, quotas.

1 Introduction

At approximately 2.5 million squire kilometres of ocean, ranging over 30 degrees of latitude, New Zealand's main exclusive economic zone (EEZ) is the fourth largest in the world and is fourteen times larger than its land mass. The fishery industry makes an important contribution to New Zealand's economy and is the fourth largest foreign exchange earner, worth NZ$1.7 billion in 2004. Around 26 000 people are directly or indirectly employed in the fishery industry (New Zealand official yearbook, 2004/05).

To maintain and improve these fisheries resources and their utilization, activities such as fishing, trawler scheduling, processing, and marketing are important. Each of these activities depends on the others. For example, production planning in a fish processing firm depends on a steady supply of fresh raw material from the fishing fleets. Also, to promote fresh fish and top quality frozen fish to the consumer, the raw material has to

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be delivered to the processors and thereafter to the consumer in top quality condition. Trawler scheduling for fishing and landing plays an important role. For these reasons, there has been growing recognition that fisheries have to be viewed as a total system from the fish in water to the fish on the plate. This system includes fishing, trawler scheduling, processing, labour allocation, quota allocation, and marketing. A mathematical model which addresses trawler scheduling, processing plans, labour allocation that could be updated with information and run periodically, would aid in the decision making process. In this paper, we develop a MILP model for fishing trawler scheduling and production planning in an integrated commercial fishery of New Zealand. The aim of this model is to give a complete idea to the manager of a fishery about when and where a trawler should go for fishing, how much raw material should be landed, how much product should be produced, and how much regular and overtime labour hours are required. The end-effects-of-planning-horizon due to variability of catch rate and ways to deal with them are also discussed.

The remainder of the paper is organised as follows. In §2 we present a detailed literature review. In §3 we give a numerical illustration of the model. The model is presented in §4 and some output of a numerical example is shown. In §5 we present a sensitivity analysis on different decisions and we conclude the paper with some final remarks in §6.

2 Literature Review

In this section, we review the existing research in (i) fishing fleets and processing, (ii) co-ordination of fishing and processing and (iii) quota allocations.

2.1 Fishing fleets and processing

Most of the related literature includes models of the fluctuating fish stock, *i.e.* developing population-level models. However, this notion is not included in this paper. There are a few papers that deal with the fishing fleet. For example, Jensson (1981) presented a simulation model which analyzed fleet operation and congestion problems. The author discussed the effect of fleet operations on the total catch, on the utilization of different factories and on the different size categories of boats. Jensson (1982) presented a fleet mix model describing the fishing fleet, vessel mix and vessel allocation.

Digernes (1982) adopted an analytical approach for single vessel operations. The author expressed revenue as a function of the operations of the vessel depending on fishing time, amount of fish gear used per fishing day, catch per unit gear used and fish price. The various cost components were associated with operational factors. For example, fuel costs were expressed as a function of engine power and operating time.

However the above mentioned papers model only trawler operation for fishing, not scheduling of trawlers. Production planning at fish processing firms is a typical product mix problem. Mikalsen and Vassdal (1981) developed a multi-period LP model for one month production planning. The authors discussed a monthly production planning model for smoothing the seasonal fluctuations of fish supply. Their model was market-driven and incorporated the acquisition of raw material purchased, rather than acquired with their own fishing fleet. Jensson (1988) developed a product mix LP to maximize profit over a five day horizon of an Icelandic fish processing firm. The model determined product mix and labour allocations. The paper analyzed the production planning, market fluctuations and randomness of raw materials for fish processing firms. The author discussed his experience of real life testing. The model had approximately 160 variables, 80 restrictions and about 60 simple upper bounds.

2.2 Co-ordinating fishing and processing

Finding the fishing schedule and processing separately may lead to suboptimization of the total system, because processing depends on a steady supply of raw material from the fishing fleets to the processing firms. Jensson (1990) proposed a mixed integer linear program to solve the co-ordinated scheduling problem of trawler landings and plant operations. The paper does not consider the trawler scheduling problem. However, the production manager of a fish processing firm needs an initial schedule for trawler landings, along with the amount of raw material that each trawler lands. Gunn *et al.* (1991) studied tactical planning for a Canadian company with integrated fishing and processing. The authors formulated an LP to determine the product mix so as to maximize profit. The model included a fleet of trawlers, a number of processing plants and market requirements. Their model ignored the short-term trawler scheduling problem, the stochastic nature of operations, and the quality-time relationship which affects the value of fish products. Millar and Gunn (1992) developed a two-stage procedure for planning marketing and fishing activities for fish processing firms. Randhawa (1994) integrated an LP and a simulation for co-ordinating fishing and processing. He determined a trawler's fishing schedule and generated the quantity of catch during the fishing trip using a simulation model. He used an LP to determine the allocation of raw material and labour, mix of products, and inventory of raw material.

2.3 Quotas

To control the continuous decrease in fish supplies, the Icelandic government introduced quota regulation in 1984, and implemented it for nine main commercial species. This system was implemented for all commercial species in 1990. In 1986, New Zealand was the first country to use quotas on a broad scale in a multi-species fishery. Currently, this program applies to 32 species in 10 management areas of New Zealand. Other countries that use individual transferable quota systems include Australia, Canada, Italy, the Netherlands, Japan and South Africa. Helgason and Olafsson (1988) presented a deterministic decision support system for long and short term fisheries management of Icelandic fisheries. The authors considered the boat type and size for fishing, temporary bans, mesh size regulations, and the catch quota allocation. They also calculated the earnings and costs in the fisheries. The model computed the expected catch, economic outcome and other statistics on a year by year basis for 10 years. However, they kept the fishing fleet, and the recruitment constant.

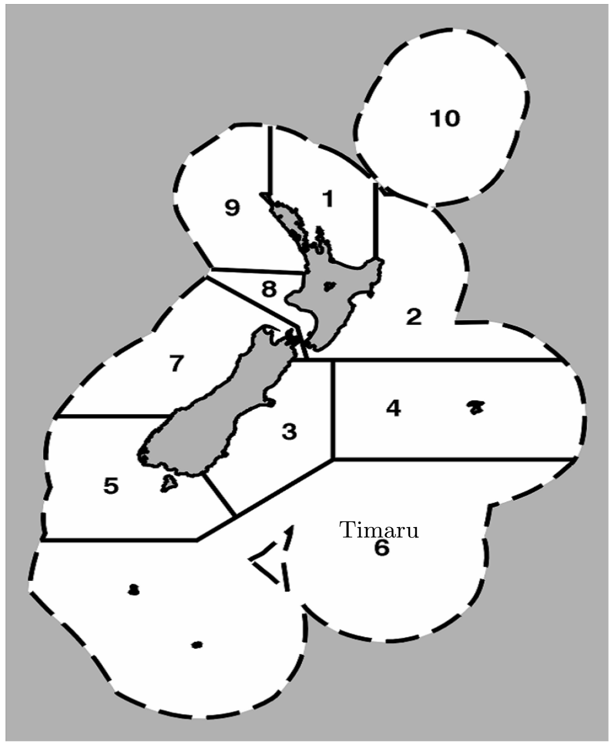

Figure 1: *Exclusive Economic Zone of New Zealand.*

3 Problem situation

We collected data from one of the major fisheries in New Zealand. The fishery has two small trawlers and one large trawler. Average expected catch for small vessels is 12 tons per day and takes two to three days per fishing trip. The average expected catch for the large vessel is 90 tons per day and takes up to 7 to 8 days per trip. The trawlers harvest 8 species over the year. In the running season, the trawlers harvest hoki, roughy, dory, ling, red cod, squid, barracouta and elephant fish. The company produces 10 different products over the year. The fish that cannot be processed during a period remain in inventory and are available for the next period production. Similarly, the product that cannot be sold during a period remains in inventory and will be sold in the next period. To allow proper testing of the model, we use some estimated numbers (for example, $FR_{i,l}$ denotes the fraction of different raw materials classified according to their quality).

In the following four subsections, we describe trawler scheduling, processing, and labour allocation.

3.1 Trawler scheduling

A trip of a fishing trawler is the movement for the purpose of fishing of the trawler from any landing port to a distinct fish stock and again from that stock to the landing port. To illustrate a trawler's trip, we show 10 species areas in the 200 miles exclusive economic zone of New Zealand, in Figure 1. For example, if a fishery is located at Timaru, then a trip can be defined as the distance travelled from Timaru to any fish stock area (say stock area 6), and returned from that stock to the fishery.

The start and end points of the trawler scheduling in our model are at the processing plant. The distance between the fishing ground and the processing plant may differ to a large extent and may impact catch preservation aspects and fuel capacity considerations. The trawler operating costs per period include the salary of the crew, diesel cost, and the average maintenance of each trawler. These costs vary according to the trawler class. Since the company owns the trawlers, the company pays the crew of the trawlers a salary. Since the trawler operation cost is fixed, we may assume that the landing price that the fishery pays to each trawler for each species and period is zero.

3.2 Processing

The processing of the fish caught by trawlers at sea usually involves chilled storage of the fish in crushed ice until the vessel returns to the port. When the trawler arrives at the freezing plant, the fish are inspected and graded by size and quality. The fish are unloaded, transported to the processing plant, and then processed according to the type and quality of the fish. At the plant, processing operations include cleaning, cutting, filleting, wrapping, skinning, forming, coating, grinding, drying, packing, and freezing. Major products include filleted, gutted, headed and gutted, dressed, fish sticks, fish blocks, *etc.* Heads, offal, *etc.* from the fish are converted to fish meal in some plants. The processing steps of different products are shown schematically in Figure 2.

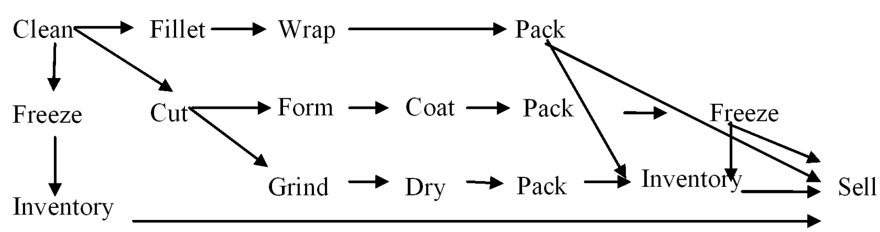

Figure 2: *Processing steps of different products.*

3.3 Labour allocation

We obtained data from the fishery regarding required labour hours per kilogram of product in different work centres for all raw materials and products, the wage rate for regular and overtime labour hours, lower and upper limits of the available labour hours, lower and upper limits of the available overtime labour hours, and the available machine hours for this fishery. Employees may work in any work centre.

4 Mathematical model

In this section, we present our mixed integer linear programming (MILP) model to coordinate fishing, processing, quota allocation and labour allocation for an integrated commercial fishery. Relevant papers discussed in §2 which may complement our ideas include the multi-period LP model of Mikalsen and Vassdal (1981) for one month production planning, the product mix LP model of Jensson (1988) to maximize profit over a five day horizon of an Icelandic fish processing firm, and the tactical planning model of Gunn *et al.* (1991) for calculating the total profit of a Canadian company with integrated fishing and processing.

We mention several differences between our formulation and the papers mentioned above. The multi-period LP model of Mikalsen and Vassdal (1981) was market-driven and incorporated the acquisition of raw material purchased, rather than acquired with their own fishing fleet. The product mix LP model of Jensson (1988) addressed labour allocation for processing firms but did not address any fleet-specific issue or the issue of quotas. The tactical planning model of Gunn *et al.* (1991) included a fleet of trawlers, a number of processing plants and market requirements. However, their model ignored trawler scheduling and labour allocation in the processing firm. Also all of the papers discussed in §2, did not take into account trawler scheduling. Since production in processing firms depends on the steady supply of raw materials from the fishing fleet, and to do so the trawler scheduling plays an important role, it is important to develop a model to address fishing trawler scheduling and processing. None of the above mentioned papers addressed trawler scheduling, processing and labour allocation comprehensively. In our MILP model, we model quota-based integrated fishery's fishing trawler scheduling for fishing, processing and labour allocation.

We define a number of indices, parameters, and decision variables separately according to fishing trawler scheduling, processing and quota allocation in Tables 1–4.

Index	Description
a	Fishing areas (fish stocks). For example, there are 10 commercial fish management areas in New Zealand
c	Work centres
i	Category of raw material by fish species and size. For example, hoki $\geq$ 50 cm, hoki $\leq$ 50 cm, cod, redfish, *etc.*
j	Type of product
l	Quality of landed fish
p	Factory
s, t, u	Periods
v	The trawler or vessel of the particular company

Table 1: *Model indices.*

Parameter	Description
A_v	Capacity of trawler v
$BIF_{i,l}$	Beginning inventory of fish i, of quality l.
$C_{a,i,t,v}$	Landing cost that the fishery pays to each vessel v of species i in stock a for each t
$E_{a,i,t,v}$	Average expected catch per day t for vessel v of species i from stock a
$ET_{a,i,u,t,v}$	Amount of fish caught, calculated according to the fishing time by subtracting the travelling and returning time from total time of a trip *i.e.* $\max\left(0, \sum_{s>u}^{t-1} E_{a,i,s,v} - TR_{a,v} \times E_{a,i,u,v} - TR_{a,v} \times E_{a,i,t-1,v}\right)$. Here $t - u \leq N_v$
$FR_{i,l}$	Fraction of the landed raw materials i of quality l
$G_{a,i}$	Quota left for species i in stock a from earlier trading and fishing, i.e $q_{a,i,0.}$
I_t	Cost of holding inventory raw materials during time t
MI	Maximum (kilograms) capacity of inventory raw materials
N_v	Maximum number of fishing days for vessel v in a planning horizon
$TR_{a,v}$	Time required to travel to fish stock a for trawlers v
$V_{t,v}$	Operating cost for vessel v on day t

Table 2: *Fishing parameters.*

Parameter	Description
$BIP_{i,j,l}$	Beginning inventory product of species i, product j, quality l
$F_{i,j}$	Fillet percentage of raw material, *i.e.* kilogram of fish species i required to produce 1 kilogram of product j
$FR_{i,l}$	Fraction of fish species i of quality l
$H_{i,j,c,}$	Working times (labour hours) required in work centre c per kilogram fish i in product j
J_t	Inventory holding cost of a product for period t
Lr	Labour cost per hour for regular time
Lo	Labour cost per hour for overtime (the overtime labour rate is 25% higher than the regular time labour rate)
LAr_t	Lower bound on regular labour hours in period t
$LR_{i,l,t}$	Lower bounds on kilograms of raw material i of quality l to be processed in period t
MIP	Storage capacity of maximum inventory product
$P_{i,j,l}$	Profit of processing plant j for raw material i, of quality l (*i.e.* the weighted net sales price of product j for raw material less all variable costs, except labour cost)
Rt	Ratio of overtime labour hours
UAr_t	Available regular labour hours in period t
$UM_{i,j,l,t}$	Upper bounds on kilograms of product j of quality l sold from raw material i for marketing reasons in period t
$UR_{i,l,t}$	Upper bounds on kilograms of raw material i of quality l to be processed in period t

Table 3: *Processing parameters.*

Variable	Description
$f_{a,i,l,t,v}$	Kilograms of fish species i of quality l from stock a landed by trawler v during period t
$q_{a,i,t}$	Quota kilograms of species i from stock a left over as available quota for period t
$r_{i,j,l,t}$	Kilograms of product j made from species i of quality l kept in inventory at the end of planning horizon t (we define $r_{i,j,l,0}$ as the initial inventory, a constant)
$s_{i,j,l,t}$	Kilograms of product j sold from raw material i of quality l during period t
$w_{p,a,u,t,v}$	1 if trawler v steams from the firm p to fish stock area a during period u for fishing and returns in period t; 0 otherwise, for all $t = 1, \ldots, T$, and all u such that $t - u \leq N_v$, and all v
$wr_{t,v}$	1 if trawler v waits in port during period t; 0 otherwise
$x_{i,j,l,t}$	Kilograms of product j produced from raw material i of quality l in period t
$yo_{t,c}$	Overtime labour hours used in work centre c during period t
yr_c	Regular labour hours used in work centre c during period t
$z_{i,l,t}$	Kilograms of fish species i kept in inventory at the end of planning horizon t (we define $z_{i,l,0}$ as the initial inventory, a constant)

Table 4: *Model decision variables.*

4.1 Objective function

Our objective is to maximize total profi, that is revenue from sales, less fishing cost, less production cost, less inventory holding cost. We therefore maximize the expression

$$\sum_i \sum_j \sum_l \sum_t P_{i,j,l} s_{i,j,l,t} - \sum_p \sum_a \sum_u \sum_t \sum_v V_{t,v} w_{p,a,u,t,v} - \sum_i \sum_l \sum_t \sum_v \sum_a C_{a,i,t,v} f_{a,i,l,t,v}$$
$$- \sum_t \sum_c Lr_t yr_c - \sum_t \sum_c Lo\, yo_{t,c} - \sum_i \sum_l \sum_t I_t z_{i,l,t} - \sum_i \sum_j \sum_l \sum_t J_t r_{i,j,l,t}.$$

4.2 Constraints

A capacity constraint denotes that trawler v can go to one fishing area a at a time, and the amount of fish caught should not exceed the capacity A_v of each trawler v. That is

$$\sum_i \sum_l f_{a,i,l,t,v} \leq A_v \qquad \text{for all } a, v \text{ and } t. \tag{1}$$

When calculating the landed fish, we employ a binary variable $w_{p,a,u,t,v}$ indicating whether a trawler goes for fishing or not. If a trawler goes for fishing, it will land its fish according to the quality of fish. Hence we have

$$f_{a,i,l,t,v} = \sum_u ET_{a,i,u,t,v} \times FR_{i,l} \times w_{p,a,u,t,v} \qquad \text{for all } p, a, i, l, t \text{ and } v. \tag{2}$$

A trawler will go to fishing or stay at port according to the requirement and profitability of the fishery. That is

$$\sum_a \sum_{t=2}^{N_v} w_{p,a,u,t,v} + wr_{1,v} = 1 \qquad \text{for all } p \text{ and } v. \tag{3}$$

The flow constraint

$$\sum_{a}\sum_{u=1}^{\max\{1,t-N_v\}} w_{p,a,u,t,v} + wr_{t-1,v} - wr_{t,v} - \sum_{a}\sum_{t1=t+1}^{\min\{t+N_v,T\}} w_{p,a,t,t1,v} = 0 \qquad \text{for all } p \text{ and } v \tag{4}$$

is also assumed, stating that if a trawler goes out for fishing, it must come back to land its catch. Bounds on raw material i of quality l in period t,

$$LR_{i,l,t} \leq \sum_{j} F_{i,j} x_{i,j,l,t} \leq UR_{i,l,t} \quad \text{for all } i, l \text{ and } t, \tag{5}$$

state that the amount of processed raw materials, should not exceed the available raw materials. An inventory balance constraint

$$z_{i,l,t-1} + \sum_{a}\sum_{v} f_{a,i,l,t,v} - \sum_{j} F_{i,j} x_{i,j,l,t} = z_{i,l,t} \qquad \text{for all } i, l \text{ and } t \tag{6}$$

indicates that fish species i of quality l which is not used for production during period t is stored as inventory ($z_{i,l,t}$) for use in the next planning horizon. Inventory storage limits for raw materials dictate that

$$\sum_{i}\sum_{l} z_{i,l,t} \leq MI \qquad \text{for all } t. \tag{7}$$

Marketing constraints on products, which indicate the amount of product sold, depends on the demands in the market. That is

$$s_{i,j,l,t} \leq UM_{i,j,l,t} \qquad \text{for all } i, j, l \text{ and } t. \tag{8}$$

An inventory products balance equation,

$$r_{i,j,l,t-1} + x_{i,j,l,t} - s_{i,j,l,t} = r_{i,j,l,t} \qquad \text{for all } i, j, l \text{ and } t \tag{9}$$

states that subtraction of the amount of product sold from the total inventory product obtained from last period and amount of product produced during the current period, yields inventory for next period. A storage capacity constraint of inventory products states that the amount of inventory products during period t should not exceed the maximum storage capacity. Hence

$$\sum_{i}\sum_{j}\sum_{l} r_{i,j,l,t} \leq MIP \qquad \text{for all } t. \tag{10}$$

The working time required for filleting, packing, freezing and stocking during period t should not exceed the total available regular and overtime, *i.e.*

$$\sum_{i}\sum_{j}\sum_{l} H_{i,j,c} x_{i,j,l,t} - yr_{t,c} - yo_{t,c} \leq 0, \qquad \text{for all } t \text{ and } c. \tag{11}$$

Bounds on the amount of regular labour available during period t take the form

$$LAr_t \leq yr_c \leq UAr_t \qquad \text{for all } t \text{ and } c. \tag{12}$$

Overtime labour should not exceed 25% of regular amount of labour. That is

$$y_{t,c} \leq Rt \times yr_c \qquad \text{for all } t \text{ and } c. \tag{13}$$

The available quota must balance over time. Hence we have a constraint of the form

$$q_{a,i,t-1} - \sum_l \sum_v f_{a,i,l,t,v} = q_{a,i,t} \qquad \text{for all } a, i \text{ and } t, \tag{14}$$

which states that subtracting the amount of fish landed from the total available quota, we calculate the remaining quota for the next periods. Variables are required to be non negative, *i.e.* $x_{i,j,l,t}$, $s_{i,j,l,t}$, $r_{i,j,l,t}$, yr_c, $yo_{t,c}$, $f_{a,i,l,t,v,}$, $w_{p,a,u,t,v}$, $wr_{t,v,}$, $z_{i,l,t}$, $q_{a,i} \geq 0$. Finally, the decision variables

$$w_{p,a,u,t,v}, wr_{t,v} \in \{0, 1\} \tag{15}$$

are required to have binary values.

4.3 A sample solution

An example solution of a 10-period model is presented below. In its first trip, trawler 1 goes for fishing to area 3 in period 1 and lands its catch in period 3. Trawler 2 goes for fishing to area 3 in period 1 and lands its catch in period 4. The details of trawler scheduling are shown in Figure 3. Starting with an initial inventory of raw materials at 25 950 kg, the model yields a total profit of \$683 692 for the fishery.

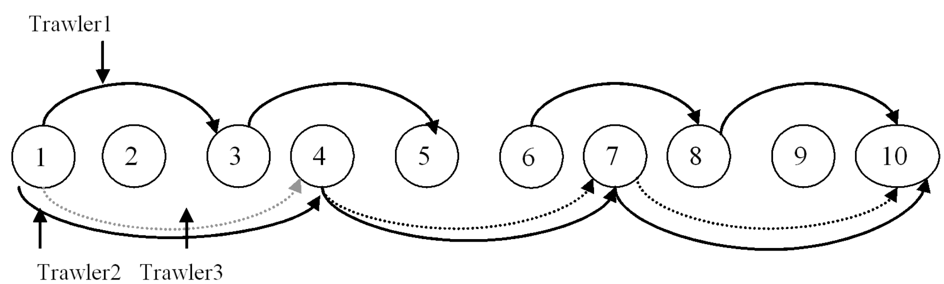

Figure 3: *A sample fishing trawler scheduling for a 10-period model.*

5 Analysis of the model

In this section, we describe a series of experiments designed to deal with the end effect of a planning horizon due to variability of catch rate, so as to observe why the model uses overtime and to observe the effect of variation of catch rate (from 10% to 50%) on profit. The aim of this sensitivity analysis is to gain knowledge about the structure and behaviour of our MILP model and to give additional information to the fishery so that it can develop guidelines for updating data and decision plans.

5.1 Safety stock for end-of-horizon effects due to variability of catch

It is common practice that managers hold safety stocks to hedge against uncertainties in supply and demand. With a deterministic model, the planning manager may use a target final inventory.

Since the catch rate of each fish species varies for different reasons (such as weather conditions and seasons), the manager of the fishery should have a safety stock at the end of each planning horizon. The safety stock of raw materials may protect against stock-out problems due to incorrect catch forecasts, weather condition, seasons, *etc.* In real life, the manager of the integrated fishery typically uses a target final inventory after the end of each planning horizon to deal with the end-of-planning-horizon effects. In our model, we set a constraint which ensures that the beginning inventory of raw materials equals the final inventory. The constraint takes the following form

$$z_{i,l,0} = z_{i,l,T} \qquad \text{for all } i, l \text{ and } t. \tag{16}$$

The model decides how much raw material will be kept as inventory at the end of a planning horizon. This is a type of safety stock which protects against variability created by the trawler schedule (man-made variability). The safety stock balances the inventory holding cost and the idle time.

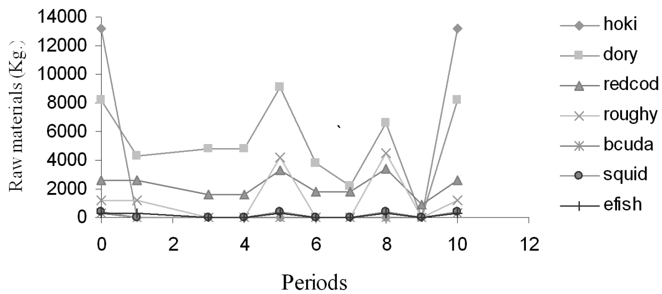

Figure 4: *Inventory raw materials of each species in different periods.*

For example, if a 10-period model is solved, we observe that the solution has a total of 80 753 kilograms of beginning and final inventory raw materials as safety stock. It yields a total profit of $742 976. The results are shown in Figure 4. It shows that the inventory raw materials vary throughout the planning horizon, but the beginning and the end inventory raw materials of each species remain the same.

The 10-period model balances inventory holding cost and uses no overtime and no idle time. However, for some planning horizons it uses overtime. For example, solving a 15-period model we notice 77 hours of overtime, in a 16-period model we observe 265 hours of overtime and in 18-period model we notice 125 hours of overtime. With the increase of planning horizon, the number of overtime hours does not increase monotonically. The number of overtime hours used by a model depends on the length of planning horizon, trawler landings and the amount of raw materials available in different periods, regular and idle hours. Considering all of these, our model first decides how much regular labour is required per period for the entire planning horizon and then decides the amount of overtimes for different periods. The details of the results are shown in Table 5.

Length of planning horizon	Regular labour per period	Overtime in the planning horizon
10-periods	998	0
14-periods	1 036	0
15-periods	1 102	77
16-periods	1 060	265
17-periods	1 117	24
18-periods	1 067	125
20-periods	1 077	210

Table 5: *Regular and overtime labour used for different planning horizons.*

5.1.1 Setting inventory naively

Instead of creating safety stock using the model, if we naively set beginning inventory raw material, the deterministic model will yield lower or higher profit depending on the amount of initial inventory raw materials. The higher the beginning inventory the higher the profit. To observe this, we considered solving the model with an initial inventory raw material of 25 950 kg. We noticed that, in period 1 the fishery uses 522 hours of labour, but in period 3 the fishery uses 1 288 hours of labour, since it achieves first landings in period 3. Our 10-period deterministic model fixed 1 030 hours as regular labour, and for each period from period 3 onwards, it uses different amounts of overtime labour for different period. We observe that in periods 3, 7 and 9 the fishery achieves two landings each. Therefore higher overtime occurs in these periods, as shown in Figure 5. The model yields a total of $683 692 profit.

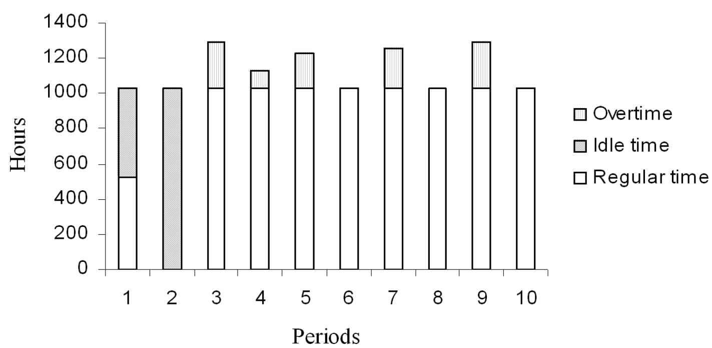

Figure 5: *Comparison of total time, regular time, overtime and idle time that occurs in model solutions.*

Using zero initial inventory raw materials, we observe that the model yields a profit of $665 518 and using 150 000 kg of initial inventory row materials (maximum storage capacity), the model yields $734 951. In §5.1 we saw that the safety stock approach yields $742 976.

Comparing the safety stock approach to naively setting inventory, we arrive at two conclusions. First, since the safety stock model calculates the beginning inventory raw materials, the profit is higher than that of naively setting beginning inventory raw materials. Second, changing the number of periods of the planning horizon for both of the above cases, we solved our deterministic model and observed that it uses some small amount of overtime. Hence it is clear that, in order to deal with the landed catch, inventory, storage capacity and market demand, a good deterministic model may use some overtime.

5.1.2 Rolling horizon approach

For the production plan, the model is solved for a number of periods and the solution is implemented. Then the model horizon is rolled forward to the point where the next decision has to be made, the new problem is solved, and the new decision is implemented. In this way, a decision plan is constructed for the true horizon of the fishery. The procedure of updating forecasts and solving the problem periodically is referred to as a rolling horizon approach. A rolling horizon approach (Blackburn and Millen (1980), Wagner and Whitin (1958)) is a strategy for managing the end effect of a planning horizon and the effect of uncertainty.

To deal with this end-of-planning horizon-effect using rolling horizon approach, the solution of a 10-period model may be repeated every five periods, since the first five periods are more certain than the last five periods. Then the parameters may be updated according to the solution obtained from the first stage and the model solved for the second stage of two stage mixed integer linear program.

5.2 Effects of moving employees among work-centre on the profit

If the employee can work in any work centre, then the fishery can reduce overtime. To verify this, we calculated the total time required to produce a product and to pack it by adding labour time in different centres for each product. We removed the work centres subscripts and modelled the work hours by means of the constraint

$$\sum_i \sum_j \sum_l H_{i,j} x_{i,j,l,t} - yr - yo_t \leq 0 \qquad \text{for all } t. \tag{17}$$

The amount of regular labour available during period t is given by

$$LAr_t \leq yr \leq UAr_t \qquad \text{for all } t. \tag{18}$$

Bounds on the amount of overtime labour are set as

$$yo_t \leq Rt \times yr \qquad \text{for all } t, \tag{19}$$

where Rt is the percentage of regular hours available for overtime hours. Overtime hours may not exceed 25% of the number of regular hour.

We notice that the model without work-centres subscripts uses 35 hours less overtime hours than the model with work-centre subscripts. We also observe that the model without work-centres subscripts yields \$742 976 which is (\$742 976 − \$741 849) = \$1 127 higher profit (0.2%) than that of the model with work-centres subscripts. We conclude that if the employees can work in any work centre, the fishery can yield slightly higher profits.

5.3 Sensitivity of profit due to variability of catch

Catch rate is the most important parameter which influences the trawler schedule and processing. Considering the last twenty years' catch rates (Clement, 2004), we determined the average expected catch for our MILP model. To observe the effect of catch variability on the profit, we used a normal distribution with five different coefficients of variation from 10% to 50% in order to generate five groups of catch rate parameters. For each coefficient of variation, we solved the model ten times and calculated the profit. The results are shown in Table 6. The percentage change in profit is defined as

$$100 \times \frac{\text{Deterministic solution} - \text{Solution with coefficient of variation}}{\text{Deterministic solution}}.$$

Here the deterministic solution yields a profit of \$742 976 which is obtained from §5.1. We notice a general decrease in the profit as variability of catch rate increases. However, the loss in profit is not monotonic. We also notice that the average profit lost for these five groups of catch rates is 4.15%, which is approximately a \$30 000 average decrease in profit.

Coefficient of variation	0.1	0.2	0.3	0.4	0.5
	Profit				
Run number	$\$ \times 10^5$	$\$ \times 10^5$	$\$ \times 10^5$	$\$ \times 10^5$	$\$ \times 10^5$
1	7.41	7.12	7.11	5.68	5.64
2	7.52	7.37	7.27	7.48	6.86
3	7.46	7.46	7.33	7.57	7.46
4	7.31	7.09	7.16	7.59	7.02
5	7.4	7.16	7.51	6.16	7.06
6	7.33	7.19	6.4	7	6.43
7	7.59	6.86	6.85	6.6	6.81
8	7.51	7.34	7.04	7.03	6.98
9	7.44	7.53	7.3	7.02	7.73
10	7.32	7.51	7.37	6.73	6.95
Average of 10 runs	7.429	7.263	7.134	6.886	6.894
% profit loss	0.01%	2.25%	3.98%	7.32%	7.21%

Table 6: *Effect of variability of catch on profit.*

5.4 Computation times

We implemented our model using the AMPL modelling language (Fourer *et al.*, 1993) and used CPLEX to solve it. Varying the number of periods of the planning horizon from 8 to 20, we solved our model on a computer with an Intel Pentium III processor with a clock speed of 665 MHz and 384 MB of RAM. Table 7 shows the computation times, number of variables and number of constraints associated with each planning horizon length.

Periods of the planning horizon	Computation time (seconds)	Variables	Constraints
8	5	3 513	3 040
9	5	3 965	3 409
10	10	4 423	3 778
11	15	4 887	4 147
12	19	5 357	4 416
13	16	5 833	4 885
14	34	6 315	5 254
15	43	6 803	5 623
19	723	8 815	7 099
20	950	9 333	7 468

Table 7: *The computational times, number of variables and number of constraints associated with each planning horizon length.*

6 Conclusion

In this paper we developed a MILP model to schedule fishing trawlers and to plan production for an integrated commercial fishery. The model co-ordinates trawler scheduling, fishing, catch quota allocations, processing and labour allocation of fisheries. The output of the model suggests when and where a trawler should go for fishing, how much raw materials should be landed, what amount of product should be produced and how much regular and overtime labour hours are required. Changing different parameters such as beginning inventory raw materials, end effect of planning horizon, overtime, idle time and work centre, we also carried out a range of sensitivity analysis. The fishery can develop guidelines for updating data and decision plans in the light of new information obtained from this sensitivity analysis. So considering the decision environment in which our MILP model is to be implemented, the role of this sensitivity analysis is important. We also discussed end-effect-of-planning-horizon due to variability of catch rate and ways to deal with them. We hope that, given the complexity of the fishery problem and the level of uncertainty in the catch rate, the MILP model will provide an efficient approach to address the decisions to be made by the fishery.

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Appendix 3

In Appendix 3, we present our paper on DBP entitled “A decomposition-based pricing method for solving a large-scale MILP model for an integrated fishery” as published in the Journal of Applied Mathematics and Decision Science (*JAMDS*).

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Research Article

A Decomposition-Based Pricing Method for Solving a Large-Scale MILP Model for an Integrated Fishery

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Recommended by Stefanka Chukova

We study the integrated fishery planning problem (IFP). In this problem, a fishery manager must schedule fishing trawlers to determine when and where the trawlers should go fishing and when the trawlers should return the caught fish to the factory. The manager must then decide how to process the fish into products at the factory. The objective is to maximize profit. We have found that IFP is difficult to solve. The initial formulations for several planning horizons are solved using the AMPL modelling language and CPLEX with branch and bound. The IFP can be decomposed into a trawler-scheduling subproblem and a fish-processing subproblem in two different ways by relaxing different sets of constraints. We tried conventional decomposition techniques including subgradient optimization and Dantzig-Wolfe decomposition, both of which were unacceptably slow. We then developed a decomposition-based pricing method for solving the large fishery model, which gives excellent computation times. Numerical results for several planning horizon models are presented.

1. Introduction and literature review

Modern commercial fisheries are often vertically integrated, that is, a firm may own fishing trawlers and a processing factory. To maximise profit, a fishery manager must schedule the fishing trawlers to determine when and where the trawlers should go fishing and when the trawlers should return the caught fish to the factory. Given a trawler schedule, the manager must then decide how to process the fish into products at the factory. The objective is to maximise profit. The difficult part of this problem is coordinating trawler scheduling and fish-processing.

Wide-ranging research has been reported on fisheries. Many papers described biological models, and only a few discussed production-planning. Mikalsen and Vassdal [1] developed a multiperiod LP model for production-planning over a one month horizon for smoothing the seasonal fluctuations of fish supply. Their model was market-driven and incorporated the acquisition of raw material purchased, rather than acquired with their owned fishing fleet. Jensson [2] developed a product mix LP model to maximize profit of an Icelandic fish-processing firm over a five-period planning horizon. He addressed production-planning and labour allocation for that processing firm but did not address any fleet-specific issue or quota issue. Gunn et al. [3] developed a model for calculating the total profit of a Canadian company with integrated fishing and processing. Their model included a fleet of trawlers, a number of processing plants, and market requirements. However, their model ignored the trawler scheduling and labour allocation in the processing firm. Indeed, none of these papers discussed models that integrated both trawler scheduling and production.

We previously developed a model [4] for the integrated fishery problem (IFP). The IFP is designed to coordinate trawler scheduling and processing and to allocate labour. It can be updated and run periodically to aid in a manager's decision making. We experimented with real data from a New Zealand fishery. Unfortunately, for realistic planning horizons of 20 periods or more, the computational times for the IFP were quite long.

To find an effective solution method, in this paper we report on our work with subgradient optimisation (SO) and Dantzig-Wolfe decomposition (DWD). Despite experimenting with different alternatives for SO and DWD, we found that these algorithms are ineffective. We then developed an effective decomposition-based pricing (DBP) method.

The remainder of the paper is organized as follows. In Section 2, we briefly present the IFP in matrix notation and its LP relaxation. Section 3 describes SO with two different Lagrangean relaxations. Section 4 describes the DWD, also with two different relaxations. Neither SO nor DWD proved effective. Section 5 gives our decomposition-based pricing procedure. While technically a heuristic, we found decomposition-based pricing to be quite efficient. As DBP was developed for linear programs, we modified it for the fishery model. We conclude the paper in Section 6 with a discussion of decomposition-based pricing to other problems and future work.

2. The fishery model

In this section, we briefly describe our fishery model [4] in matrix notation.

Parameters.

(i) c^1, c^2, c^3 denote unit profit of trawler operation, raw fish inventory, and fish-processing, respectively,

(ii) A^0 denotes quantity of fish landed per trip in each period,

(iii) D^1 denotes mass balance coefficients on each trawler in each period,

(iv) D^2 denotes mass balance coefficients on fish within the processing factory,

(v) A^1, A^2 denote mass balance coefficients governing transformation of raw fish into a finished product.

Decision variables.

(i) w denotes binary variables indicating whether a trawler takes a given trip,

(ii) f denotes raw fish inventory, indicating the current quantity of each type of raw fish in each period,

(iii) x denotes fish-processing variables, indicating that a given type of raw fish is converted into a given product.

$$\begin{aligned}
\text{Model IFP:} \quad & \text{maximize} && c^1 w + c^2 f + c^3 x \\
& \text{subject to} && \\
\text{Inventory supply constraints} \quad & && A^0 w + f = 0, && (2.1)\\
\text{Trawler scheduling constraints} \quad & && D^1 w = b^1, && (2.2)\\
\text{Processing constraints} \quad & && D^2 x = b^2, && (2.3)\\
\text{Inventory demand constraints} \quad & && A^1 f + A^2 x = b^0, && (2.4)\\
& && w \in \{0,1\}, && (2.5)\\
& && f, x \geq 0. && (2.6)
\end{aligned}$$

Equation (2.1) represents the relationship of the trawler-scheduling variables w to landed fish f, as a mass balance in movement of fish from trawlers to the factory. Equation (2.2) expresses the constraints involving only trawler scheduling, indicating, for example, that a trawler may be in only one place at a time. Equation (2.3) expresses fish-processing constraints, modelling the flow of fish through the factory as raw fish is made into various products. Equation (2.4) represents the mass balance constraints, representing the flow of raw landed fish inventory into the fish-processing factory.

When the integer constraints (2.5) are relaxed, the model is the usual linear programming relaxation. However, other relaxations are possible. Observe that IFP decomposes into a trawler-scheduling problem and a fish-processing problem if either constraint set (2.1) or constraint set (2.4) were relaxed. In the next section, we use both decompositions with SO.

3. Subgradient optimisation for the fishery model

Lagrangean relaxation is based on the existence of complicating constraints. When these complicating constraints are relaxed, the resulting model is often easier to solve. Geoffrion [5] introduced the term "Lagrangean relaxation," developed relevant theories, and explored its usefulness for IP branch and bound. Fisher [6] reviewed Lagrangean relaxation and documented a number of successful applications of this method. To obtain the Lagrangean relaxation of IFP, we attach multipliers θ to complicating constraints of IFP, and bring this term into the objective function. SO is a commonly used method of finding the optimal multipliers θ (Held et al. [7], Held and Karp [8], and Shepardson and Marsten [9]). This approach yields θ directly. In this section, we describe our attempts to solve IFP with SO, with two different decompositions.

Table 3.1. Numerical results for SO, relaxing constraint set (2.4).

	5 periods	10 periods	30 periods
IP optimum	\$522 764	\$1 065 775	\$2 300 871
LP optimum	\$522 764	\$1 066 350	\$2 331 036
SO optimum	\$522 764	\$1 065 991	\$2 325 650
SO solution time (s)	718	1120	3625

3.1. Relaxation of inventory balance constraints. In this section, we use SO to solve the fishery model by relaxing the complicating inventory balance constraints (2.4)

$$\begin{aligned} \mathrm{PR1}(\theta): \underset{f,w,x}{\mathrm{Max}} \{ & c^1 w + c^2 f + c^3 x - \theta(A^1 f + A^2 x - b^0) \mid A^0 w + f = 0, \\ & D^1 w \le b^1,\ D^2 x \le b^2 \}. \end{aligned} \tag{3.1}$$

The SO algorithm for IFP can be stated as follows. Denote θ^k as the Lagrangean multiplier at iteration k.

Step 1. Initialize iteration $k = 0$ and set jump size t^k. θ^0 was taken from the dual values of constraint set (2.4) from the LP relaxation.

Step 2. Solve $\mathrm{PR1}(\theta)$ for θ^k.

Step 3. Let $\theta^{k+1} = \theta^k + t^k(A^1 f + A^2 x - b^0)$.

Step 4. Set $t^{k+1} = t^k(0.9998) * (\text{Lagrangean value} - \text{LP value})/\text{slack}$, where slack = slack + $(A^1 f + A^2 x - b^0)^2$.

Step 5. For convergence:
if $|\theta^{k+1} - \theta^k| < \varepsilon$, then stop,
else if the maximum number of iterations was reached, then stop,
else let $k = k+1$ and go back to Step 1.

Table 3.1 shows numerical results for models with various different planning horizons.

3.2. Relaxation of landed fish constraints. We next attempted SO by relaxing constraint set (2.1) as follows:

$$\mathrm{PR2}(\theta): \underset{f,w,x}{\mathrm{Max}} \{ c^1 w + c^2 f + c^3 x - \theta(A^0 w + f) \mid D^1 w \le b^1,\ D^2 x \le b^2,\ A^1 f + A^2 x = b^0 \}. \tag{3.2}$$

Numerical results of various planning horizon models are shown in Table 3.2.

SO was ineffective in both of these decompositions, taking far too long to converge. Subgradient optimization has been reported to result in unpredictable convergence behaviour (Guignard and kim [10]) and such was the case with this model. We experimented with modifications to update the Lagrangean multipliers, but this decreased computational time only slightly. We therefore turned to Dantzig-Wolfe decomposition.

TABLE 3.2. Comparison between LP and LR relaxation solutions and true optimum (IP).

	5 periods	10 periods
IP optimum	\$522 764	\$1 065 775
LP optimum	\$522 764	\$1 066 350
LR optimum	\$522 764	\$1 070 450
SO solution time (s)	952	1360

4. Dantzig-Wolfe decomposition (DWD) for the IFP

In this section, we apply DWD (Dantzig [11]). DWD yields θ as the dual variable associated with the relaxed constraints. This decomposition may be interpreted in the following way: the fishery manager uses a master model to generate prices for raw fish. These prices are passed to the fishing-trawler captains who propose trawler schedules, and to the factory manager who proposes a production schedule. Their proposals are passed to the fishery manager, who uses the master model to find the best mix of proposals and new prices for raw fish. The procedure terminates when no new proposals come from the subproblems.

Algebraically, we express the feasible region of the trawler-scheduling subproblem as a convex combination of the extreme points for constraint sets (2.1) and (2.2). Since the trawler-scheduling variables are bounded, this set is bounded. Similarly, we can express the feasible region of the production-planning subproblem and constraint set (2.3) as a convex combination of its extreme points. Without loss of generality, these variables are bounded, so their convex set is bounded. Let λ^1 and λ^2 be variables associated, respectively, with the subproblems for trawler scheduling and fish-processing, with extreme points numbered $1,\dots,K1$ and $1,\dots,K2$. We can then write the DWD master problem as follows:

$$\text{Maximize} \sum_{k=1}^{K1} \lambda^{1k}\left(c^1 w + c^2 f\right) + \lambda^{2k} \sum_{k=1}^{K2} c^3 x, \quad \text{subject to}$$

$$\text{Inventory balance rows} \quad \sum_{k=1}^{K1} \lambda^{1k}\left(A^1 f\right) - \sum_{k=1}^{K2} \lambda^{2k}\left(A^2 x\right) = 0, \tag{4.1}$$

$$\begin{aligned} &\text{Trawler scheduling} && \sum_{k=1}^{K1} \lambda^{1k} = 1, \\ &\text{Fish-processing} && \sum_{k=1}^{K2} \lambda^{2k} = 1, \quad \lambda^{1k}, \lambda^{2k} \geq 0. \end{aligned} \tag{4.2}$$

Note that λ^{1k} is continuous, so this model will only provide an upper bound.

Let θ be the dual prices associated with the inventory balance constraint (4.1). The subproblems are as follows.

(1) Trawler-scheduling subproblem S_1^k,

$$\begin{aligned}&\text{maximize } c^1 w + c^2 f - \theta(A^1 f), \quad \text{subject to}\\&\text{constraint sets (2.1) and (2.2),}\\&f \geq 0,\ w \in \{0,1\}.\end{aligned} \tag{4.3}$$

(2) Processing subproblem S_2^k,

$$\begin{aligned}&\text{maximize } c^3 x - \theta(A^2 x), \quad \text{subject to}\\&\text{constraint set (2.3),}\\&x \geq 0.\end{aligned} \tag{4.4}$$

We used AMPL [12] to solve IFP with DWD on a Pentium3 computer. The results were really quite disastrous. Giving the master a good initial feasible solution, a small five-period model required 1168 iterations and 4 hours, 54 minutes. Giving the master an initial feasible solution of the zero vector, the five-period model required 1068 iterations and 3 hours, 59 minutes, but a direct solution with CPLEX takes only four seconds to solve a five-period model directly. We further attempted to use DWD to solve models with more periods, but these took a very long time to solve.

We next tried to solve IFP by relaxing the landed-fish constraint set (2.1). This proved even worse computationally. A trivial three-period model required 1367 iterations with a naive initial solution, and 1787 iterations with an initial solution of the zero vector. A five-period model required 3923 iterations, and a ten-period model was abandoned after 4536 iterations.

We conclude that Dantzig-Wolfe decomposition is not effective for IFP.

5. Decomposition-based pricing (DBP)

In this section, we apply decomposition-based pricing (DBP) for the efficient solution of IFP. Mamer and McBride [13] developed DBP for multicommodity flow problems. As with DWD, subproblems are created by dualizing some constraints, and these subproblems are identical to S_1^k and S_2^k from the DWD. Instead of using the subproblem to produce an extreme point of the relaxed polytope for inclusion in a master problem, the optimal basic columns of the subproblem are included in a restricted master. The DWD master is replaced by a version of the original problem with all of the original rows and a subset of original columns. This restricted master problem is solved to obtain an improved primal solution and new dual prices. The restricted master is not the same as the DWD master. It has a full IFP formulation with restricted rows. The procedure terminates when no positive variables entered into the restricted master or when the objective value of the subproblems and that of the restricted master are equal. Dual prices from the inventory balance constraints (2.4) are passed to the subproblems.

The fishery problem is an MILP model, so we cannot guarantee strong duality (outside of a custom branch and bound algorithm). Hence this is a heuristic method. However,

Table 5.1. LP relaxation solution and IP solution of different planning horizon models.

Planning horizon	Variables in original problem	LP objective value	IP objective value
5	2193	\$522 764	\$522 764
10	4423	\$1 066 350	\$1 065 775
15	6803	\$1 607 944	\$1 582 008
20	9333	\$1 898 411	\$1 880 196
25	11 989	\$2 141 757	\$2 121 887
30	16 139	\$2 331 037	\$2 300 871

through a careful choice of initial feasible solution and stopping criteria, we obtain excellent bounds, and the solution times obtained are much faster than the direct solutions with CPLEX.

5.1. DBP procedure for the fishery model. We use Lagrangean relaxation to relax the inventory balance constraints (2.4), as in Section 3. Let θ be the simplex multipliers for the restricted master where θ is associated with the inventory balance constraints (2.4). We define the restricted master as the original problem for IFP, but restricted to a smaller set of variables I^k. Set I^k is the set of positive variables in the master at iteration k. Set I^k increases in size with each iteration because each iteration of the subproblems adds new variables to I^k. Computationally, we found (as did Mamer and McBride [13]) that the number of variables in I^k at any iteration is much less than the number of variables in the original problem:

$$\begin{aligned}(M^k)\text{maximize } c^1w + c^2f + c^3x, &\quad \text{subject to}\\ &\text{constraint sets (2.1) to (2.4)},\\ &f \geq 0, \quad w \in \{0,1\}, \quad x \geq 0,\end{aligned} \tag{5.1}$$

with $f, w, x \in I^k$, here I^k is the index set of all positive variables $f, w, x \geq 0$.

Our decomposition-based pricing procedure is summarised as follows.

Step 1. Initialize. Set iteration $k = 1$. We used three alternate methods to pick an initial set of prices θ^1.

(I1) Start with $\theta^1 = 0$.

(I2) Start with θ^1 as the dual prices from the relaxed constraints of the IFP LP relaxation.

(I3) Start with heuristic dual prices, $\theta^1_{ilt} = -\sum_{j:Fij>0} P_{ij}/(2.5 \cdot F_{ij})$, where $F_{i,j}$ is the fillet percentage of raw material and $P_{i,j,l}$ is the profit of processing product j of quality l from raw materials i.

Step 2. Solve subproblems S_1^k and S_2^k. For each f_i, w_i, or $x_i > 0$, add the variable to I^k. Thus, $I^k = \{f_i, w_i, x_i > 0 \text{ in } S_1 \text{ or } S_2 \text{ for any iteration } 1, 2, \ldots, k\}$.

Step 3. Solve the restricted master M^k and get dual prices θ^k.

Table 5.2. Numerical results for dbp under different initial dual prices and stopping criteria.

Solution method	Planning horizon	Iterations	Solution time(s)	Variables in final master	DBP solution	Solution gap (%)
I1-SC1	5	26	156	1308	\$522 764	0.00%
I1-SC1	10	29	257	2815	\$1 065 775	0.00%
I1-SC1	15	32	341	4272	\$1 579 440	0.16%
I1-SC1	20	29	365	5691	\$1 874 097	0.32%
I1-SC1	25	29	414	7026	\$2 119 938	0.09%
I1-SC1	30	25	544	8115	\$2 293 803	0.31%
I1-SC2	5	29	211	1252	\$522 764	0.00%
I1-SC2	10	30	258	2576	\$1 065 538	0.02%
I1-SC2	15	32	335	3881	\$1 579 309	0.17%
I1-SC2	20	27	348	5065	\$1 870 047	0.54%
I1-SC2	25	29	557	6253	\$2 118 528	0.16%
I1-SC2	30	31	1,737	7324	\$2 288 997	0.52%
I2-SC1	5	27	192	1356	\$522 764	0.00%
I2-SC1	10	33	292	2873	\$1 065 531	0.02%
I2-SC1	15	30	322	4378	\$1 579 321	0.17%
I2-SC1	20	28	496	5874	\$1 864 368	0.84%
I2-SC1	25	27	433	7135	\$2 117 990	0.18%
I2-SC1	30	32	1,042	8277	\$2 266 274	1.50%
I2-SC2	5	28	208	1282	\$522 764	0.00%
I2-SC2	10	28	252	2724	\$1 065 712	0.01%
I2-SC2	15	35	373	4092	\$1 579 466	0.16%
I2-SC2	20	29	359	5420	\$1 875 597	0.24%
I2-SC2	25	35	534	6540	\$2 111 616	0.48%
I2-SC2	30	30	650	7623	\$2 292 894	0.35%
I3-SC1	5	26	178	1325	\$522 764	0.00%
I3-SC1	10	32	275	2784	\$1 065 775	0.00%
I3-SC1	15	30	312	4130	\$1 579 447	0.16%
I3-SC1	20	31	351	5524	\$1 876 023	0.22%
I3-SC1	25	32	487	7135	\$2 120 282	0.08%
I3-SC1	30	27	613	8052	\$2 295 376	0.23%

Step 4. For stopping criterion, we used two alternate methods.

(SC1) Stop when the objective values of the subproblem and restricted master are equal, $v(S_1^k + S_2^k) = v(M^{k+1})$. Here, we solve the trawler-scheduling subproblem as an LP. By solving this subproblem as LP, we find good variables to add to the restricted master, with fast computation time.

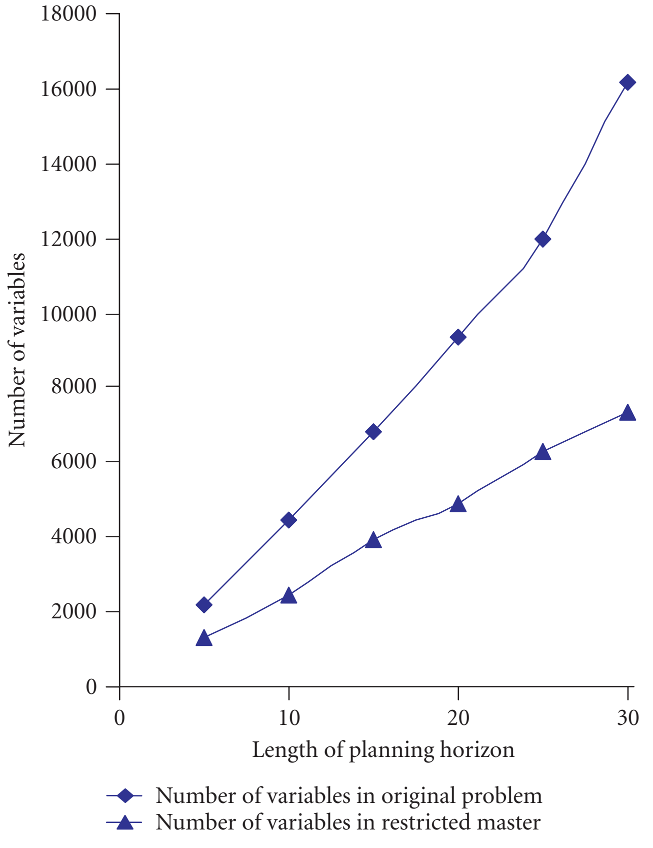

FIGURE 5.1. Comparison of the number of decision variables in DBP and that of IP.

(SC2) Stop when no new variables come into the restricted master problem. Here, we solve the trawler-scheduling subproblem as an IP.

Else go to Step 2.

Step 5. After the LP optimum is found, solve the final restricted master problem as an IP.

Table 5.1 shows the optimal LP and IP objective values. Depending on the initial feasible solution and stopping criterion, we ran the DBP algorithm in five different ways: I1-SC1, I1-SC2, I2-SC1, I2-SC2, and I3-SC1, as shown in Table 5.2.

The objective values obtained from our DBP procedure are very close to the optimal solutions. The best method, I3-SC1, had an average percentage solution gap of only 0.12%. Thus DBP takes far fewer iterations and much less time than DWD.

Figure 5.1 shows that the number of variables in the final DBP restricted master is typically fewer than half the number of original variables.

6. Conclusion

In this paper, we described our work with relaxation and decomposition techniques for the IFP. We found that both subgradient optimization and Dantzig-Wolfe decomposition were ineffective, under either of two different decompositions. Finally, we applied decomposition-based pricing to IFP. Our decomposition-based pricing procedure was the most effective method by far. We used real data from a commercial fishery, but the work has not been implemented in the fishery operation as yet. Using DBP, we see no impediment to implementation. More importantly, we believe that DBP can be adapted for other integer programs. Future work includes continuing to explore ways to improve the efficiency of DBP for integer programs.

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Appendix 4

In Appendix 4, we present our paper on DBONP and RCBP methods, entitled "Two pricing methods for solving an integrated commercial fishery planning model," as accepted for publication in the South African Journal of Operational Research (*ORiON*).

Two Pricing Methods for Solving an Integrated Commercial Fishery Planning Model

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Abstract

In this paper, we develop two novel pricing methods for solving an integer program. We demonstrate the methods with an integrated fishery. In that problem, a fishery manager must (1) schedule fishing trawlers, to determine when and where the trawlers should go fishing, and when the trawlers should return the caught fish to the factory. The manager must then (2) decide how to process the fish into products at the factory. The objective is to maximise profit.

The problem may be modeled as a single integer program, with both the trawler scheduling and production planning parts integrated, in an integrated fishery planning model (IFPM). Inventory constraints connect the two parts of the problem. Production planning alone would be an easy linear program, but due to the trawler scheduling aspect, IFPM is a difficult integer program. For IFPM, traditional solution methods result in computation times that are far too long to be practical.

The two pricing methods developed in this paper are (i) a decomposition based O'Neill pricing (DBONP) method and (ii) a reduced cost based pricing (RCBP) method. We demonstrated the methods by numerical examples for different planning horizons, corresponding to different sized problems. Results are very positive.

Key words: Decomposition; pricing; reduced cost; fishing trawler scheduling; processing.

1. Introduction

This paper presents recent research on the solution of an integer program for an integrated commercial fishery's activities. A modern commercial fishery has two loosely-connected problems to solve. The first is to schedule trawlers for fishing, including deciding where and when those trawlers should work, and, crucially, when they should return to land the fish. The landed fish generally becomes inventory, which is raw material for a processing plant. The processing plant cleans, processes, and packages the fish for the market.

Based on data for a commercial fishery here in New Zealand, we previously developed a model (Hasan and Raffensperger, 2006) to solve this problem: the integrated fishery planning model (IFPM). IFPM is designed to co-ordinate trawler scheduling and processing. The model could theoretically be updated and run periodically to aid in a manager's decision making, and we used real data from a New Zealand fishery. Unfortunately, for realistic planning horizons of 20 periods or more, computational times for IFPM were quite long, even with methods such as Dantzig-Wolfe decomposition and subgradient optimisation, so we set out to find better means to solve it.

We have developed two novel column generation algorithms to solve IFPM. These algorithms are based on the decomposition-based pricing algorithm of Mamer and McBride (2000), combined with the integer variable pricing method of O' Neill *et al.* (2005).

1.1. The fishery planning literature

Wide-ranging research has been reported on fisheries. Many papers described biological models, but only a few discussed production planning. Mikalsen and Vassdal (1981) developed a multi-period linear programming (LP) model for one month production planning for smoothing the seasonal fluctuations of fish supply. Their model was market-driven and incorporated the acquisition of raw material purchased, rather than acquired with their own fishing fleet. Jensson (1988) developed a product mix LP model to maximize profit of an Icelandic fish processing firm over a five period planning horizon. He addressed production planning and labour allocation for that processing firm but did not address any fleet-specific issue or quota issue.

Gunn *et al.* (1991) developed a model for calculating the total profit of a Canadian company with integrated fishing and processing. Their model included a fleet of trawlers, a number of processing plants and market requirements. However, their model ignored the trawler scheduling and labour allocation in the processing firm. Indeed, none of these papers discussed models that integrated both trawler scheduling and production.

1.2. The integer programming literature

Integer programming has received quite a lot of attention in the literature. We describe a few papers that have informed our work here.

Martin *et al.* (1985) presented a reduced cost based branch-and-bound method for solving mixed integer programs. The author formulated two candidate problems on the basis of 0 or 1-integer variables and then optimized both of the candidate problems to get the MILP solution.

Mamer and McBride (2000) developed a decomposition-based pricing (DBP) procedure for LPs. Their algorithm works by solving subproblems, just as the Dantzig-Wolfe algorithm uses subproblems, but the DBP master is in the same form and structure as the original model, but with far fewer variables. Variables that are positive in the subproblem are brought directly into the master; all other variables are omitted from the master. That paper is key background to this one. DBP has also been used by V. de Carvalho (2006) for cutting stock, and Raffensperger & Schrage (2007) for scheduling training for a tank battalion. We have previously applied DBP for IFPM Hasan and Raffensperger (2007). In this paper, we describe two improved DBP methods.

The first method we call *decomposition based O'Neill pricing* (DBONP), because it is based on the work of O' Neill *et al*. (2005). O'Neill *et al*. (2005) developed a technique for constructing a set of linear prices from solving a MILP and an associated LP, based on a theorem of Gomory and Baumol (1960). They first solved a MILP, set the integer variables to their optimal values, and then removed the integrality constraints to convert the MILP to an LP. They used the dual prices obtained from this LP to form an efficient contract (the dual of *IFPML*) in the context of an electricity market.

The second method is a reduced cost based pricing (RCBP) method. Unlike Martin *et al.* (1985), we set constraints for both 0 and 1- integer variables (O'Neill *et al*., 2005) in the same candidate problem, which is the restricted master in the proposed RCBP method. In this method, we do not solve a subproblem at all. Instead, we choose new variables for the restricte master based on a reduced cost calculation, and we bring a group of variables in the restricted master problem at each iteration. We will see that, both of these methods produce better solutions than our earlier work (Hasan and Raffensperger, 2007).

The remainder of this paper is organized as follows. In Section 2, we briefly present the IFPM. In Section 3, we review O'Neill's pricing method and discuss the mathematical formulation of the proposed DBONP method. We also present the DBONP algorithm along with numerical examples. In Section 4, we discuss the mathematical formulation of the proposed RCBP method, and also present the RCBP algorithm along with numerical examples. Section 5 compares the solutions obtained from DBONP and RCBP method with DBP. Section 6 concludes the paper.

2. The fishery model in matrix notation

In this section, we briefly describe our IFPM in matrix notation. The details of the model can be found in Hasan and Raffensperger (2006). We have omitted details of the model in order to focus on the algorithm, and for the sake of brevity, we plead the reader's forbearance with annoying details of superscripts.

Parameters

c^1, c^2, c^3, unit profit of trawler operation, raw fish inventory, and fish processing, respectively,

A^0, quantity of fish landed per trip in each period,

D^1, mass balance coefficients on each trawler in each period,

D^2, mass balance coefficients on fish within the processing factory,

A^1, A^2, mass balance coefficients governing transformation of raw fish into finished product.

Decision variables

w, binary variables indicating whether a trawler takes a given trip,

f, raw fish inventory, indicating the current quantity of each type of raw fish in each period,

x, fish processing variables, indicating that a given type of raw fish is converted into a given product.

Model IFPM: maximize $c^1w + c^2f + c^3x$, subject to

Inventory supply constraints	A^0w	$+f$		$=0$	(1)
Trawler scheduling constraints	D^1w			$=b^1$	(2)
Processing constraints			D^2x	$=b^2$	(3)
Inventory demand constraints		A^1f	$+A^2x$	$=b^0$	(4)
Integrality of the trawler variables	$w \in \{0,1\}$				(5)
Nonnegativity of inventory and production		$f,$	$x \geq 0.$		(6)

Equation (1) represents the relationship of the trawler scheduling variables w to landed fish f, as a mass balance in movement of fish from trawlers to the factory. Equation (2) expresses the constraints involving only trawler scheduling, indicating, for example, that a trawler may be in only one place at a time. Equation (3) expresses fish processing constraints, modelling the flow of fish through the factory as raw fish is made into various products. Equation (4) represents the mass balance constraints, representing the flow of raw landed fish inventory into the fish processing factory. When the integer constraints (5) are relaxed, the model is the usual linear programming relaxation.

IFPM consists of trawler scheduling and processing, connected by inventory constraints, either (1) or (4). Using Lagrangean relaxation, we can relax either of these side constraints, and the model would decompose into an integer program for the trawler scheduling, and a linear program for the fish processing. These separate problems would be easier to solve, and the sum of their objective values will be an upper bound (since it is a maximization problem) on the optimal value of IFPM. For example, if we relax (4), we obtain the following two subproblems, where θ is the vector of dual prices on (4):

$$PR1_\theta\text{: Max } c^3x - \theta A^2x\text{: } D^2x \leq b^2,\ x \geq 0. \tag{7}$$

$$PR2_\theta\text{: Max } c^1w + c^2f - \theta A^1f\text{: } A^0w + f = 0,\ D^1w \leq b^1,\ w \in \{0,1\}, f \geq 0. \tag{8}$$

Now, we have said θ is the vector of dual prices on (4), but that assumes that IFPM is solved as an LP, not as an IP. It is too bad that $PR2_\theta$ cannot be directed by some kind of price information on the integer variable w. In fact, the DBONP method actually finds such price information, and uses it.

Following the decomposition-based pricing method for this problem (Hasan and Raffensperger, 2007), the master would follow from the original problem, with its same structure, and all its constraints. However, it would begin initially with only enough variables to allow a feasible solution. In IFPM, the zero vector is feasible, as the fishery manager can simply choose to do nothing.

At each iteration k, the master M^k is solved as a linear program, in order to find the necessary dual prices θ. These prices are passed to the subproblems $PR1_\theta$ and $PR2_\theta$, which are then solved. Positive variables from the subproblems are then passed to the master, increasing the total number of variables that it contains. The set of variables in the master is tracked by the indices of the variables found thus far, in an index set I^k. (As we are using matrix notation, it should be understood that the index set is at the vector component level.) Thus, a variable with its index in I^k has been positive in a subproblem in some previous iteration, and will appear in the master. Variables that have always been zero in every subproblem do not have their index in I^k, and thus do not appear in the master.

In matrix-vector notation, M^k can be written as

M^k: Max $c^1 w + c^2 f + c^3 x$, subject to constraint sets (1) to (4),

$f, w, x \geq 0$, with $f, w, x \in I^k$, where I^k is the index set of positive variables found in the subproblems,

$f, w, x = 0$, for $f, w, x \notin I^k$.

While this approach is better than direct integer programming with CPlex, we wished to improve this method further.

3. Decomposition-based O'Neill pricing (DBONP)

In this section, we first discuss the O'Neill pricing in Section 3.1. In Section 3.2, we present the mathematical formulation of the DBONP. In Section 3.3, we present the DBONP algorithm and in Section 3.4 we discuss numerical examples of different planning horizons.

3.1. O'Neill's pricing method

O'Neill *et al.* (2005) developed a technique for constructing a set of linear prices from solving a MILP and an associated LP, based on a theorem of Gomory and Baumol (1960). O'Neill *et al.* were not interested in solution as such, nor in computation time, but in finding efficient prices for indivisible objects.

To find these prices, they first solved a MILP to optimality. They then add new constraints that fix the integer variables to their optimal values, and then removed the integrality constraints to convert the MILP to an LP. Solution of this model gave dual prices to the new constraints. They showed that the dual variables in the associated LP have a traditional interpretation as prices. The dual variables explicitly price integral activities, and clear the market in the presence of nonconvexities. They used these dual prices to form an efficient contract in the context of a market for electricity.

We reproduce the Gomory & Baumol theorem here for completeness.

Theorem 1: A MILP with *m* continuous variables and *n* integer variables $\left(R^m \times Z^n\right)$ that has a feasible and bounded optimal solution can be converted to an LP with at most $(m + n)$ continuous variables R^{m+n} and at most *n* additional linear constraints.

3.2. Mathematical formulation for DBONP

To apply the method of O'Neill *et al.* (2005) with DPB, our method

(1) solves the restricted master as an integer program;

(2) fixes the integer variables to their optimal values with new constraints,

$$w = w^*, \tag{9}$$

and solves the restricted master as an LP, thus obtaining dual price information $\theta 1$ on the $w = w^*$ constraints;

(3) then uses the resulting dual prices $\theta 1$ to better inform the trawler scheduling subproblem as to which variables should be selected. The trawler subproblem can use this new information through Lagrangean relaxation of the new constraints:

$$PR2_{\theta,\theta 1}\text{: Max } c^1w + c^2f - \theta A^1 f - \theta 1(w - w^*)\text{: } A^0 w + f = 0,\ D^1 w \leq b^1,\ w \in \{0,1\}, f \geq 0. \quad (10)$$

(4) Positive variables from both subproblems are brought into the restricted master. Stopping criteria is when no new positive variables are produced, or when the objective values of the subproblems and master are equal. By explicitly pricing the integer variables, and using that price information in the subproblem, we bring better variables into the restricted master, back in step (1).

Note, however, that this approach requires solving the restricted master as an integer program at every iteration. This is computationally burdensome. We therefore use ordinary DBP, solving the restricted master and subproblems as LPs, until we find the LP optimum. We then switch to the formal DBONP method, and continue iterating. This creates two separate loops. The first loop does not use the $w = w^*$ constraints; the second loop does.

Loop 1. Relax the inventory balance constraint (4), and then apply DBP, to obtain the final restricted master as an LP.

Step 0: Initialize. Set iteration $k = 1$. Set the initial prices $\theta^1 = 0$.

Step1: Solve subproblems $PR1_\theta$ and $PR2_\theta$, with $PR2_\theta$ as IP. For $w^i > 0$ put i in I^k, where $I^k = \{i\text{: } w^i > 0 \text{ in } PR1_\theta, \text{ and } PR2_\theta \text{ for any iteration } 1,2,\ldots,K\}$.

Step 2: Solve M^k as an LP and get dual prices θ^k and pass them to the subproblems.

Step 3: If $v(PR1_\theta + PR2_\theta) = v(M^{k+1})$, then go to Loop 2. Else go to step 1.

Loop 2. Solve the current restricted master as an IP, and add constraints which fix the integer variables to their optimal values. We solve the master as an LP and obtain the dual prices on the inventory balance (4) constraint, and the equations associated with the integer variables. We have the dual prices θ^k as before, but now we also have new dual prices $\theta 1$ from the new constraints.

Step 4: Solve the restricted master problem as an IP.

Step 5: For integer variables, fix $w^i = w^{i*}$.

Step 6: Solve master with fixed w^i as LP. Obtain dual prices θ^k and $\theta 1$, and pass them to the subproblems.

Step 7: Solve the subproblems $PR1_\theta$ and $PR2_{\theta,\theta 1}$ with the dual prices obtained from step 6. If no new variables enter into the restricted master, then stop. Else go back to step 4.

We present a flowchart of DBONP in Figure 1.

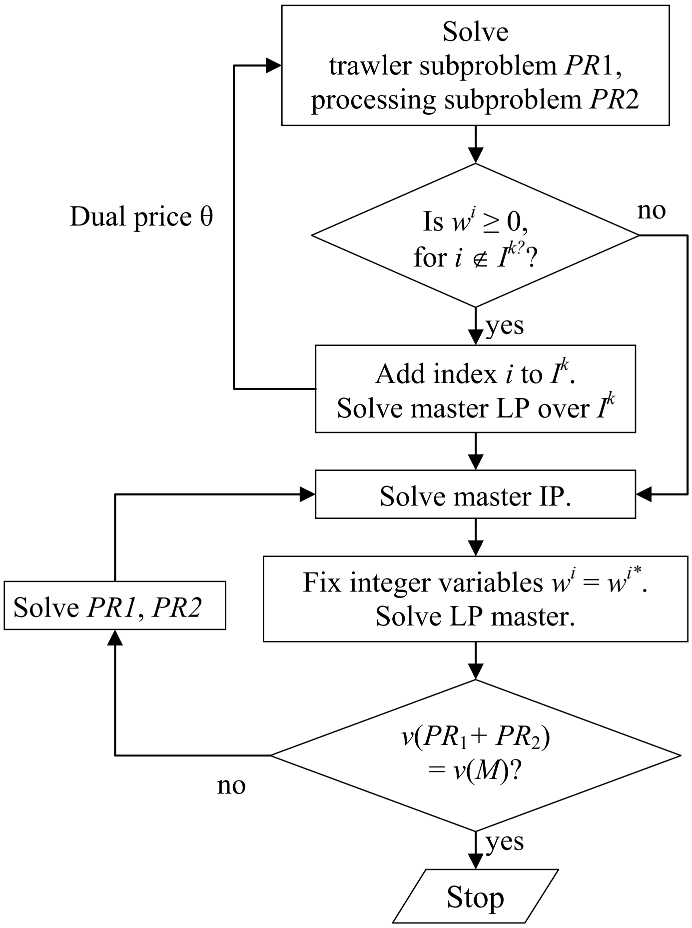

Figure 1: Flowchart of DBONP

3.3. Numerical results

We compare the solutions of DBONP with those obtained from the original IFPM, LP relaxation, and DBP. Results are presented in Table1. We observe no duality gap for 5, 10 and 25-period models, thus confirming optimality. But the 15, 20, and 30-period models have slight gaps. For example, a 30-period model has only 0.02% solution gap. The average percentage solution gap of six different planning horizon models is only 0.04%. These gaps can be considered trivial indeed.

Length of planning Horizon	Number of variables	Number of Iterations	Solution time (s)	DBP solution	DBONP solution	% solution gap
5	489	29	217	$522,764	$522,764	0.00%
10	1,284	27	216	$1,065,540	$1,065,775	0.00%
15	2,229	33	345	$1,579,309	$1,579,570	0.15%
20	3,324	48	912	$1,874,097	$1,878,580	0.08%
25	6,440	45	796	$2,120,282	$2,121,887	0.00%
30	6,938	44	3562	$2,293,803	$2,300,230	0.02%

Table 1: Comparison of the optimal solutions obtained from DBP and DBONP methods.

Naive creation of initial dual. Notice that we started with dual prices of θ=0. Instead, we tried creating the initial dual prices naively. Results are reported in Table 2. Solutions obtained from DBONP are close to the true optima. The average % gap is only 0.06%, but sometimes worse than the table above.

Planning horizon	Number of Variables	Number of Iterations	Solution time (s)	DBP solution	DBONP solution	% solution gap
5	1,264	29	208	$522,764	$522,764	0.00%
10	2,601	30	266	$1,065,540	$1,065,540	0.02%
15	4,087	36	387	$1,579,309	$1,580,670	0.08%
20	4,926	50	1045	$1,874,097	$1,873,950	0.30%
25	6,259	43	710	$2,120,282	$2,121,887	0.00%
30	8,277	50	3129	$2,293,803	$2,300,460	0.01%

Table 2: Comparison of the number of iterations, time and solutions obtained from DBP and DBONP.

Tables 1 and 2 show that the solutions obtained from DBONP are either equal to or very close to the optimal solutions (15-period, 20-period and 30-period models). To see why this little difference in profit remains, we compared the true optimal solution with that from DBONP of a 30-period planning horizon problem. The total number of trawler trips in DBONP coincides with that of the original problem, but the schedule is slightly different, as shown in Figures 2 and 3. As a result, there is a slight change in the processing and holding costs.

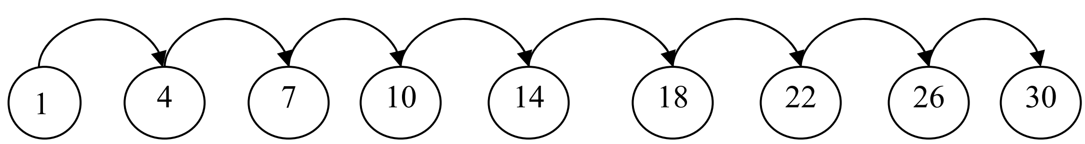

Figure 2: Trawler 1 schedule in the optimal solution.

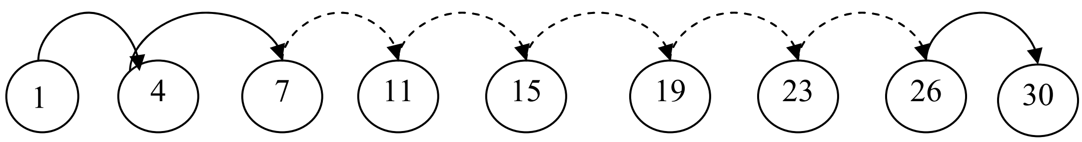

Figure 3: Trawler 1 schedule in the DBONP solution.

Figures 4, 5, and 6 show the solution times, % gap, and number of iterations, for different planning horizon models, when solved by DBP and DBONP. DBONP takes a higher number of iterations and more computation time, but produces better solutions than that of DBP.

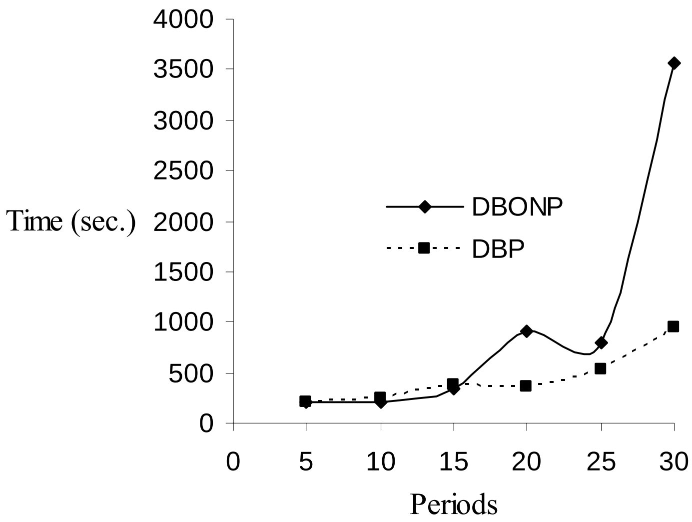

Figure 4: Solution times for different planning horizons by DBP and DBONP.

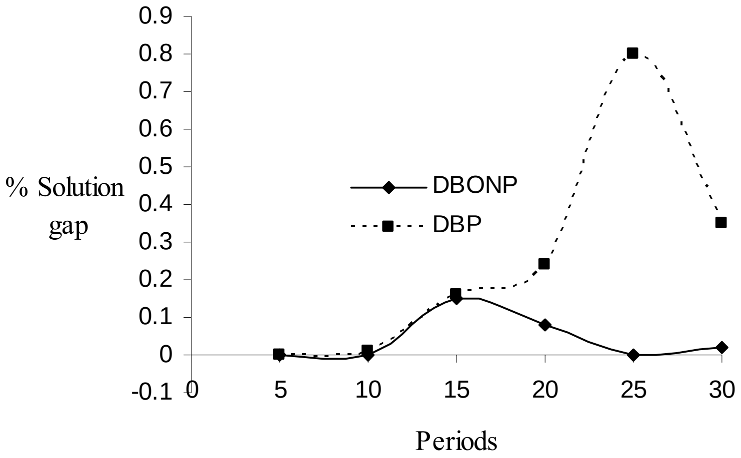

Figure 5: % solution gap of DBP and DBONP.

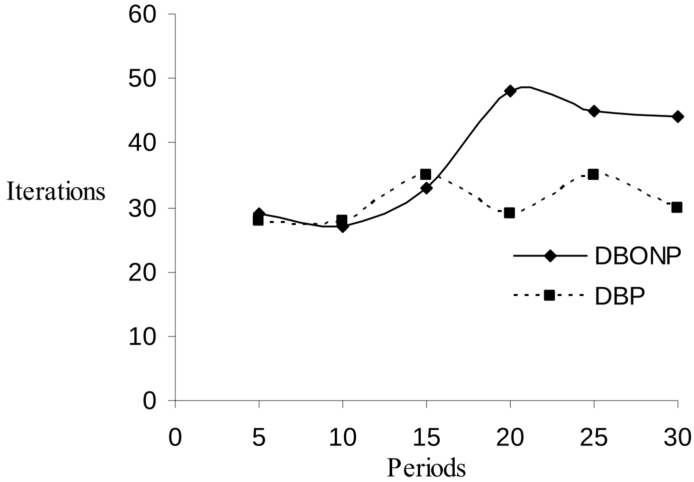

Figure 6: Number of iterations required to solve different planning horizons by DBP and DBONP.

In this section, we used both DBP and O'Neill pricing to develop the DBONP technique. We found that DBONP had slightly longer computation times, but produced better solutions than our earlier DBP procedure. To improve the computation times further, we next develop a reduced cost based pricing method.

4. Reduced Cost Based Pricing for IFPM

One reason that the DBONP algorithm took a relatively long computation time was due to solution of the trawler scheduling subproblem as an integer program. What if we eliminated this step? Instead, we can use the O'Neill price information to find the reduced cost for each integer variable. We are moving away from the DBP philosophy for the trawler scheduling aspect of the problem, but we will continue to use DBP for the fish processing subproblem. So the processing subproblem, and the restricted master, are the same as with DBP. But instead of using the trawler scheduling subproblem, we just calculate the reduced cost of the variables of that subproblem, which is extremely fast.

4.1. Reduced cost of a variable

The reduced cost of a variable w_j with objective function coefficient c_j is the net change in the objective function to generate one unit of w_j, and is defined as $\bar{c}_j = c_j - z_j$. The reduced cost gives the marginal value of a variable on the objective function related to the current basic solution. For a maximization problem, the variable with largest positive reduced cost will be the incoming variable. Following the notation in AMPL (Fourer *et al.*, 1993), we will denote the reduced cost of variable w as $w.rc$. Denote λ^1 and λ^2 as the dual prices on (1) and (2) respectively, with a^0 and d^1 as the relevant columns of A^0 and D^1 respectively.

$$w.rc = c^1 - \lambda^1 a^0 - \lambda^2 d^1 - \theta 1 \tag{11}$$

This reduced cost calculation has an explicit term for the integrality constraint. Next, we show how to use this reduced cost calculation.

4.2. RCBP algorithm

Step 0. Initialize, with k=1. Solve M^1.

Step 1. Solve M^k as an IP.

Step 2. Add constraints (9) for the integer variables $w = w^*$. Solve the restricted master as an LP. Obtain dual prices for the trawler scheduling constraints (1), (2), and (9).

Step 3. Calculate $w.rc$, (11). Scan the reduced costs for all integer variables. Include integer variables with positive reduced cost to the restricted master. For the continuous variables from the fish processing part of the problem, we have two options:

> Option 1: All continuous variables appear in every restricted master.
>
> Option 2: Continuous variables with positive reduced cost are added to the restricted master at each iteration.

Step 4. For the processing subproblem, solve the processing LP subproblem, and add all positive variables to the restricted master as with DBP.

Step 5. If no new variable enters the restricted master, then stop. Else go back to step 1.

We present a flowchart of RCBP in Figure 7.

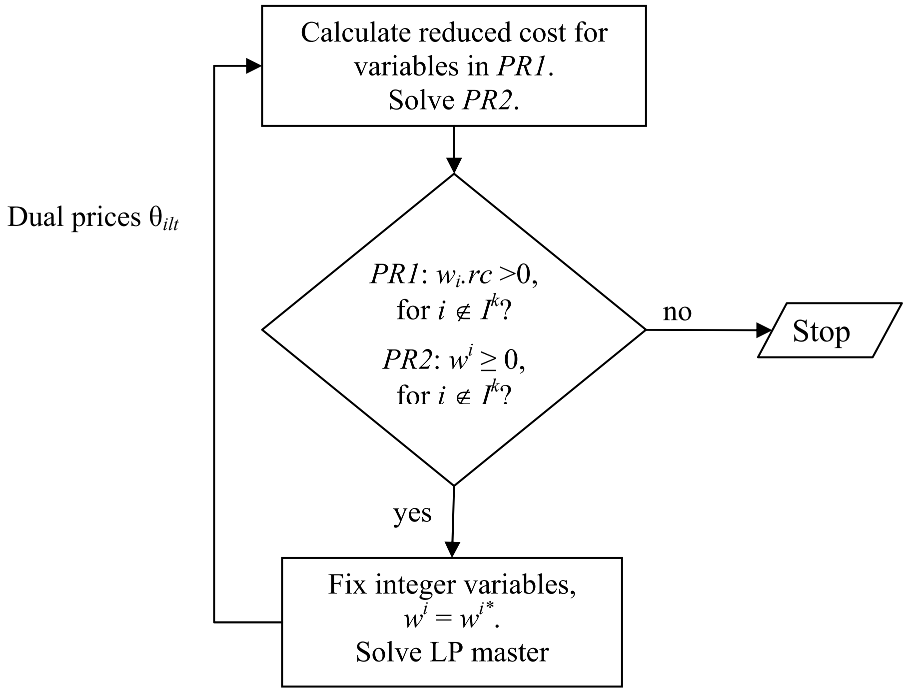

Figure 7: Flowchart of RCBP.

4.3. Numerical results

We solved IFPM with different planning horizon models using each option in Step 3. Option 2 takes fewer iterations and less time to solve the fishery model than does Option 1. Results are reported in Table 3.

Planning Horizon	Description of entering variables	Number of iterations	Solution time (Sec.)	RCBP optimal value	% Solution gap
5	Option 1	5	39	$522,764.0	0
	Option 2	3	5	522,764.0	0
10	Option 1	10	142	1,065,538.0	0.02
	Option 2	5	15	1,065,538.0	0.02
15	Option 1	11	113	1,582,006.9	0
	Option 2	5	53	1,582,008.4	0
20	Option 1	7	109	1,877,275.0	0.15
	Option 2	4	71	1,879,928.0	0.01
25	Option 1	6	74	2,107,736.0	0.66
	Option 2	8	111	2,121,887.0	0
30	Option 1	8	262	2,284,545.0	0.71
	Option 2	10	901	2,299,648.0	0.05

Table 3: Total profit, iterations, and solution time from RCBP procedure.

5. Comparison of DBP, DBONP and RCBP

We compare the optimal solutions, number of iterations, and solution times obtained from decomposition based pricing (DBP), decomposition based O'Neill's pricing (DBONP) and reduced cost based pricing (RCBP) in Figures 8 to 10. RCBP is the best among the methods we developed. It takes the least time to solve, requires fewer iterations and yields better solutions. We further investigated these two methods using three different problem instances, under many different catch rate scenarios. The numerical results here are consistent with those from the other problem instances.

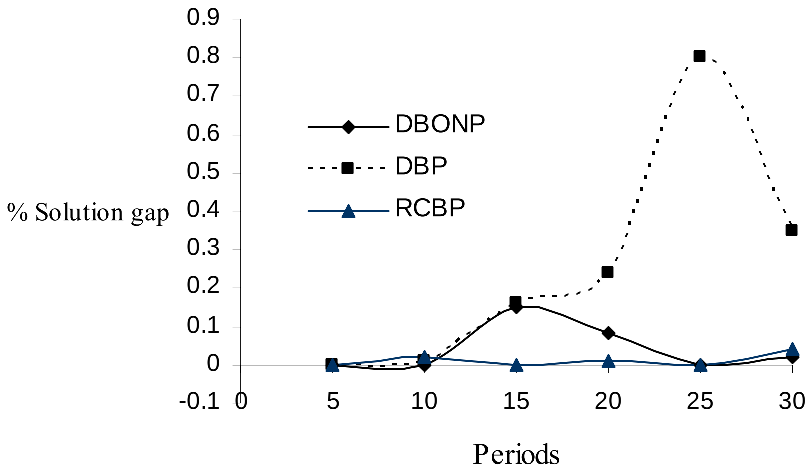

Figure 8: % solution gap of DBP, DBONP, and RCBP.

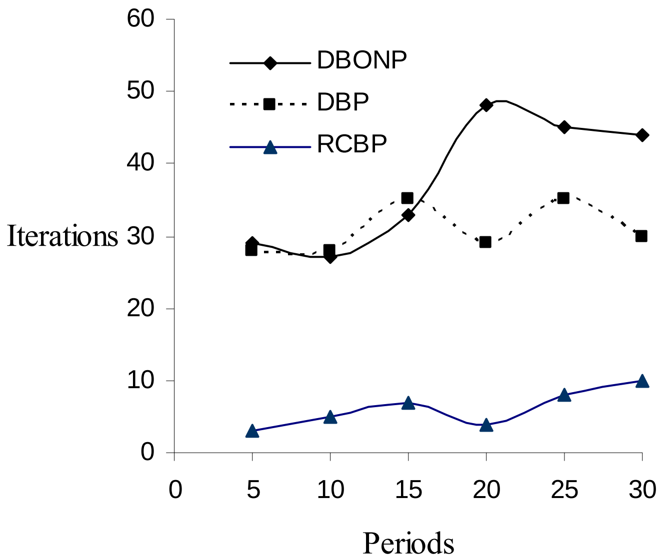

Figure 9: Number of iterations to solve DBP, DBONP, and RCBP.

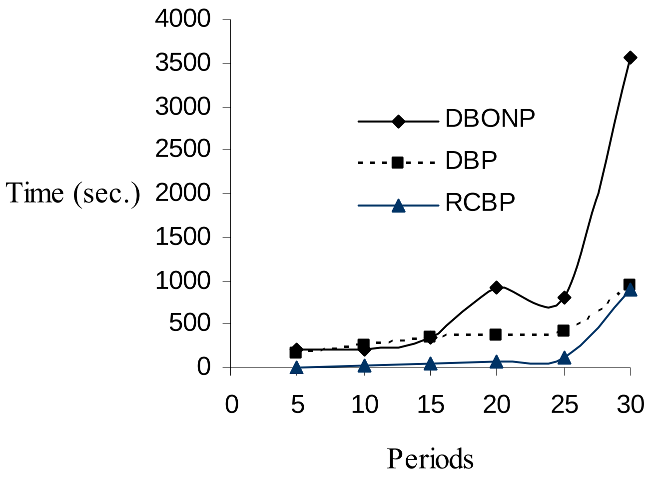

Figure 10: Solution times to solve DBP, DBONP, and RCBP.

6. Conclusion

In this paper, we developed two different column generation algorithms for a faster solution of IFPM. The first was decomposition-based O'Neill pricing (DBONP). The second was a reduced cost based pricing (RCBP), also based on O'Neill pricing.

In RCBP, we solved only easy LP subproblems and avoided the need to solve the integer subproblem. Instead of solving the IP trawler scheduling subproblem, we calculated the reduced cost for each variable, choosing variables with positive reduced cost to bring into the restricted master.

Compared to DBP alone, we found that the DBONP algorithm took slightly longer, but tended to produce better solutions. However, RCBP is both faster and gives better solutions than DBP, and in some cases than the DBONP method.

Note that we never employed specialized branch and bound, except for that native to CPlex in the restricted master and the subproblem. It therefore appears that the combination of DBP and O'Neill pricing may allow a range of new column generation algorithms that could prove effective for integer programs.

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Appendix 5

In Appendix 5, we present our paper on rolling horizon, entitled “How good is the rolling horizon approach for an integrated fishery planning model?” as accepted for publication in the International Journal of Ecological Economics and Statistics (IJEES).

How good is the Rolling Horizon Approach for an Integrated Fishery Planning Model?

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Abstract

In this paper, we apply a traditional decomposition method of rolling horizon for reducing the problem size for solving an integrated fishery planning model (IFPM). The purpose of the paper is to examine the value of using rolling horizon approach for planning and implementing fishery plans. From the experiments with different problem instances, we find that the classic approach of a rolling horizon was ineffective in the sense that it took long time to solve the 30-period IFPM, and reduced profits significantly.

Another purpose of this paper is to demonstrate the capacity of the IFPM to assist the fishery in performing sensitivity analysis on some important model parameters. A fishery manager employing these models would be motivated to ask "what-if" questions upon the observance of the model solutions. Considering the decision environment in which the IFPM is to be implemented, the role of sensitivity experiments is very important. Numerical results for different problem instances are presented.

Keywords: Integer programming; trawler scheduling; fish processing; rolling horizon; end-effect.

1 Introduction and Literature Review

Quota-based integrated commercial fisheries own fishing trawlers, processing plants, and fish quotas. To maintain and improve these fisheries resources and their utilization, activities such as fishing, trawler scheduling, labour allocation, processing and marketing are important. Each of these activities depends on the others. For example, production planning in a fish processing firm depends on a steady supply of fresh raw material from the fishing fleets. Also, to promote good quality products to the consumer, the raw material has to be delivered to the processors in good quality condition. Trawler scheduling for fishing and landing plays an important role.

In this paper, we examine a rolling horizon approach for reducing the size of the problem and to deal with the end-of-planning-horizon effects for solving the IFPM developed in Hasan & Raffensperger (2006). We also present some sensitivity analysis of the IFPM. Considering the decision environment in which the IFPM is to be implemented, the role of sensitivity experiments is very important. A fishery manager employing these models would be motivated to ask "what-if" questions upon the observance of the model solutions. The purpose of this paper is to demonstrate the capacity of the IFPM to assist the fishery in performing sensitivity analysis on some important model parameters.

To illustrate sensitivity analysis, some of the more critical model parameters such as trawler capacity, available quotas, available labour etc were varied, and their effect on the model output were investigated. The aim of this sensitivity analysis is to help the fishery to develop guidelines for updating data and decision plans in the light of new information obtained from this sensitivity analysis.

Unlike popular problems in the OR literature, standard test problems for our model do not exist. Modifying one real-model data set, we generate three more different problem instances. Care however must be exercised in order to generate instances with feasible solutions.

Mikalsen and Vassdal (1981) developed a multi-period linear programming (LP) model for a one month production planning model for smoothing the seasonal fluctuations of fish supply. That model was market-driven and incorporated the acquisition of raw material purchased, rather than acquired with their owned fishing fleet. Jensson (1988) developed a product mix LP model to maximize profit of an Icelandic fish processing firm over a five period planning horizon. He addressed production planning and labour allocation for that processing firm but did not address any fleet-specific issue or quota issue. Gunn, Millar and Newbolt (1991) developed a tactical planning model for calculating the total profit of a Canadian company with integrated fishing and processing. Their model included a fleet of trawlers, a number of processing plants and market requirements. However, their model ignored the trawler scheduling and labour allocation in the processing firm. Millar (1998) analyze the impact of rolling horizon planning on the cost of industrial fishing activity. The author analyzed the rolling horizon planning for a MILP model which only addresses the fishing trawler scheduling of an integrated fishery.

This paper significantly extends Hasan and Raffensperger (2006). That paper gave a mixed integer linear program (MILP) to model trawler scheduling, processing plans, and labour allocation. The model can be updated and run periodically to aid in the decision making process, over some finite planning horizon. Due partly to inability to solve large MILP models, and partly to inability to forecast catch and demand, the planning horizon is necessarily short.

To overcome the deterioration of catch and demand forecast for a long horizon, managers often use a smaller model horizon than any reasonable estimate of the firm's future horizon. This results in end-of-planning-horizon effects, which are suboptimal solutions arising from the use of a short model horizon. In particular, deterministic MILP models tend to leave zero inventory raw materials in the final period. To manage this problem, rolling horizon approaches have been widely used to overcome these problems, especially in material requirement planning systems (Blackburn & Millen,1980, Wagner & Whitin, 1958, Fisher et al, 2001). The integrated fisheries received little attention in these areas by the researchers.

The remainder of this paper is organized as follows. In section 2, we briefly illustrate the model. In Section 3, we present the structure of the model, computation time and number of variables in different planning horizon models. In Section 4, we present the rolling horizon approach. We also analyse the impact of quota allocations on the profit and also compare the average profit of different planning horizon models. We analyse the impact of the capacity of the trawler on the profit. And in Section 5, we describe the direction of future research and conclude.

2 Fishery Environment

The data used for the experiments are obtained from a major fishery in New Zealand. The fishery has a fleet of three trawlers. Two of the trawlers are small vessels which catch an average of 12 tons per day, take two to three days per fishing trip, and can go up to 21 days. The third trawler catches an average of 90 tons per day, takes 7 to 8 days per trip, and can go up to 60 days. The trawlers harvest 8 species over the year. In the running season, the trawlers harvest hoki, roughy, dory, ling, red cod, squid, barracouta and elephant fish. The company produces 10 different products over the year. For the fishery model, we consider a 10 period planning horizon. The fish that cannot be processed in a period remain in inventory and are available for the next period production. Similarly, the product that cannot be sold in a period remains in inventory and will be sold in the next period. In the following four subsections, we briefly describe quotas, trawler scheduling, processing, and labour allocation.

2.1 Catch Quota

A permit to fish is a specified amount of a quota stock in a given period, usually a year. The quotas can be issued for free, against a fee, or at a public auction to companies or individual vessels. In case the quotas are issued and not auctioned off, the allocation is based upon a specified reference called the *quota base*. To control the continuous decrease in fish supplies, the Icelandic government introduced quota regulation in 1984, and implemented it for nine main commercial species. This system was implemented for all commercial species in 1990. In 1986, New Zealand was the first country to use quotas on a broad scale in a multi-species fishery. Currently, this program applies to 32 species in 10 management areas of New Zealand. Other countries that use individual transferable quota systems include Australia, Canada, Italy, the Netherlands, Japan and South Africa.

2.2 Trawler scheduling

A trip of a fishing trawler is the movement for the purpose of fishing from any landing port to a distinct fish stock, and again from that stock to the landing port. The trawler operating costs per period include the salary of the crew, diesel cost, and average maintenance of each trawler. These costs vary according to the trawler class. Since the company owns the trawlers, the company pays the trawlers a salary. Since the trawler operation cost is fixed, we may assume that the landing price that the fishery pays to each trawler for each species and time period is zero.

2.3 Processing

When a trawler arrives at the freezing plant, the fish are inspected and graded by size and quality. The fish are unloaded, transported to the processing plant, and then processed according to the type and quality of the fish. At the plant, processing operations include cleaning, cutting, filleting, wrapping, skinning, forming, coating, grinding, drying, packing, and freezing. Major products include filleted, gutted, headed and gutted, dressed, fish sticks, fish blocks, etc. Heads, offal, etc., from the fish are converted to fish meal in some plants.

2.3 Labour allocation

The fishery under study provided the required labour hours per kilogram of product in different work centres for all raw materials and product, the wage rate for regular and overtime labour hours, the lower and upper limit of the available labour hours, the lower and upper limit of the available overtime labour hours, and the available machine hours for this fishery. Employees may work in any work centre.

3. Structure of the IFPM and computation times

In this section, we present the structure of the IFPM and discuss the computational difficulty for solving the longer planning horizon models.

3.1 Structure of the model (IFPM)

The IFPM consists of a trawler scheduling and a processing subproblem along with complicating side constraints containing variables from both the subproblems. And hence, it is hard to solve.

Here we present the structure of the model in matrix-vector notation.

Parameters

c^1, c^2, c^3, unit profit of trawler operation, raw fish inventory, and fish processing, respectively,

A^0, quantity of fish landed per trip in each period,

D^1, mass balance coefficients on each trawler in each period,

D^2, mass balance coefficients on fish within the processing factory,

A^1, A^2, mass balance coefficients governing transformation of raw fish into finished product.

Decision variables

w, binary variables indicating whether a trawler takes a given trip,

f, raw fish inventory, indicating the current quantity of each type of raw fish in each period,

x, fish processing variables, indicating that a given type of raw fish is converted into a given product.

IFPM: maximize $c^1w + c^2f + c^3x$,

subject to

Inventory supply constraints, $A^0w + f = 0.$ (1)

Trawler scheduling constraints, $D^1w = b^1.$ (2)

Processing constraints $D^2x = b^2.$ (3)

Inventory demand constraints, $A^1f + A^2x = b^0.$ (4)

$w \in \{0,1\}$ (5a)

$f,\ x \geq 0.$ (5b)

Equation (1) represents the relationship of the trawler scheduling variables w to landed fish f, as a mass balance in movement of fish from trawlers to the factory. Equation (2) expresses the constraints involving only trawler scheduling, indicating, for example, that a trawler may be in only one place at a time. Equation (3) expresses fish processing constraints, modelling the flow of fish through the factory as raw fish is made into various products. Equation (4) represents the mass balance constraints, representing the flow of raw landed fish inventory into the fish processing factory.

3.2 Test problems generation

In this section, modifying the original problem data, we extract three more different test problems. These three problems are referred to as “IFPMS,” “IFPML,” and “IFPMXL”. The “IFPMS” is smaller than the original problem. It has fewer trawlers and quality types. The “IFPML” is larger than the original problem. It has a higher number of trawler and stock areas. “IFPMXL” has a higher number of trawlers and stock areas than the other problems. A summary of these problems for the IFPM is given in the Table 1.

Characteristics		Original Problem	IFPMS	IFPML	IFPMXL
Number of trawlers	:	3	2	4	6
Number of factories	:	1	1	1	1
Number of species	:	8	8	8	8
Number of stock areas	:	2	2	3	4
Number of quality types	:	3	2	3	3
Number of product types	:	3	4	3	3
Number of constraints	:	10,885	6,550	15,456	24,404
Number of continuous variables	:	9,685	5,785	11,533	12,981
Number of integer variables	:	2,556	1728	5,124	7,620

Table 1: A summary of four different problems of 30-period planning horizons for the IFPM.

In the following section, we discuss the solution time and difficulties for solving longer planning horizon problems.

3.3 Computation times

We implemented our model using the AMPL modelling language (Fourer *et al.*, 1993) and used CPLEX (ILOG Corp., www.ilog.com) to solve it. Varying the number of periods of the planning horizon from 5 to 30, we solved our model on computer with an Intel Pentium III processor with a clock speed of 665 MHz and 384 MB of RAM. Table 2 shows the optimal profit, computation time, number of integer and continuous variables associated with each planning horizon of 5 to 30-periods of the original problem. AMPL's presolve eliminates some variables. For example, a 30-period model has 14,699 variables. But presolve eliminates 2,458 variables and shows a total of 12,241 variables (2,556 integer and 9,685 continuous). The longer planning horizon models take considerably longer to solve. For example, we ran and abandoned a 29 and a 30-period model after more than five hours.

Planning Horizon	Solution time (sec)	$v(P)$ ($)	$v(\bar{P})$ ($)	Variables	
				Integer	Continuous
5	3	522,764	522,764	156	4,110
6	4	556,945	557,440	180	4,333
7	3	770,767	770,767	216	4,556
8	7	812,587	813,076	258	4,779
9	4	1,013,345	1,013,345	306	5,002
10	10	1,065,775	1,066,350	360	5,225
11	6	1,255,777	1,255,777	414	5,448
12	13	1,313,945	1,314,621	468	5,671
13	85	1,431,831	1,466,321	522	5894
14	28	1,506,253	1,515,077	576	6,117
15	53	1,582,008	1,607,944	630	6,340
16	60	1,621743	1,648,103	684	6,563
17	73	1,695,835	1,734,379	738	6,786
18	356	1,746,724	1,774,867	792	7,009
19	81	1,826,217	1,859,060	846	7,232
20	131	1,880,196	1,898,411	900	7,455
21	166	1,931,858	1,963,397	954	7,678
22	1354	1,962,473	1,992,527	1,008	9,701
23	1429	2,007,252	2,056,248	1,058	8,124
24	1632	2,048,128	2,084,239	1,740	8,347
25	153	2,121,887	2,141,757	1,872	8,570
26	328	2,146,273	2,173,053	2,000	8,793
27	1008	2,192,681	2,220,159	2,136	9,016
28	331	2,236,589	2,258,272	2,274	9,239
*29	16,745	2,261,176	2,295,345	2,414	9,462
*30	18,240	2,300,871	2,331,036	2,556	9,685

Table 2. IP profit, number of integer and continuous variables obtained from the solution of 5 to 30-period original problem.

* The model was abandoned after more than five hours, and so the solution shown may not be optimal.

We also tried to run the 30-period model by a computer with 1.73MHz Pentium M, with 512 MB of RAM. But we gave up after 28 hours. Windows indicated that it had run out of memory, and was trying to allocate more virtual (hard disk) memory. Thus, we can say for sure that this model would require a lot of memory.

Using the same computer we tried to solve the problems IFPMS, IFPML and IFPMXL for 5 to 30 period planning horizons. The results are shown in Table 3.

Problem	PH	Solution time (Seconds)	Variables		IP profit ($)	LP profit ($)
			Integer	Continuous		
IFPMS	5	02	50	910	335,477	335,477
	10	03	200	1,885	701,182	702,866
	15	03	450	2,860	1,123,295	1,123,295
	20	84	800	3,835	1,439,023	1,461,530
	25	324	1,238	4,810	1,705,280	1,733,364
	*30	Abandoned after 12 hours	1,728	5,785	1,874,130	1,905,126
IFPML	5	03	140	1,683	660,701	665,741
	10	15	580	3,653	1,347,194	1,353,447
	15	359	1,320	5,623	1,852,260	1,891,241
	*20	Abandoned after 5 hours	2,360	7,593	2,196,291	2,234,440
	*25	Abandoned after 5 hours	3,664	9,563	2,400,920	2,444,345
	*30	Abandoned after 5 hours	5,124	11,533	2,550,260	2,605,895
IFPMXL	5	03	210	1,806	732,706	747,420
	10	34	810	4,041	1,542,810	1,554,154
	15	36,540	1,932	6,276	1,994,834	2,006,230
	*20	Abandoned after 5 hours	3,522	8,511	2,248,057	2,262,451
	*25	Abandoned after 5 hours	5,490	10,746	2,396,554	2,416,450
	*30	Abandoned after 5 hours	7,620	12,981	2,546,817	2,568,376

Table 3: IP profit, number of integer and continuous variables obtained from the solution of 5 to 30-period IFPMS, IFPML, and IFPMXL.

* The model was abandoned after more than five hours, and so the solution shown may not be optimal.

We tried and abandoned a 30-period model of IFPMS after more than 12 hours. To solve a 20-period model of IFPML, we tried more than 5 hours and abandoned. We also attempted to solve 25 and 30-period models but failed to solve. A 15-period model of IFPMXL took 10 hours and 9 minutes to solve and yielded a total profit of $1,994,830. We also attempted to solve 20, 25 and 30-period models but these failed to solve.

Therefore from the solution times of all four different problem instances, we found that the longer planning horizon problems are hard to solve. To help the fishery to solve the IFPM efficiently, we will apply a rolling horizon approach in the following section.

4. Rolling horizon approach

In this section, we examine the value of using a rolling horizon approach for planning and implementing fishery plans. There are several reasons behind the use of this rolling horizon approach. First, to reduce the size of the problem to make it solvable. Second, to overcome the difficulty of catch and demand forecasts for a long horizon, the manager of the fishery may use a shorter planning horizon. And third, to deal with the end-of planning horizon effect.

The procedure of updating forecasts and solving the problem periodically is referred to as a rolling horizon approach. A rolling horizon approach (Blackburn & Millen (1980), Wagner & Whitin (1958)) is a strategy for decomposing a large problem to make it solvable, and for managing the end-of planning horizon effect in deterministic models. This approach has been widely used in production planning (Fisher et al, (2001)). Millar (1998) analyzed the impact of rolling horizon planning on the cost of industrial fishing activity. He analyzed the rolling horizon planning for a MILP model which addressed only the fishing trawler scheduling of an integrated fishery.

Also, due partly to the inability to solve large MILP models and partly to inability to forecast catch and demand, the planning horizon is necessarily short. To overcome the difficulty of catch and demand forecasts for a long horizon, managers may use a shorter planning horizon than any reasonable estimate of the firm's real future horizon. This results in end-of-planning-horizon effects, which are suboptimal solutions. For example, deterministic MILP models tend to leave zero inventories in the final period unless a minimum final inventory is prescribed. Because, if there is no need for the inventory holdings, the fishery will be interested to process all of the landed fish as soon as it is available and sell the products for profit. Also why pay inventory holding costs of raw fish if there is no need for final inventory.

In the following section, we present the rolling horizon algorithm for the IFPM along with numerical illustrations.

4. 1 Rolling horizon algorithm

In a rolling horizon, we want to solve for a planning horizon T. We will solve a set of models each with horizon $T2$ where $T2 << T$. We initialize the starting period $T1$ of the planning horizon to 1. We will fix and implement decisions and data for fixed horizon η where $\eta << T2$. The number of models with horizon $T2$ = round$\left(\frac{T}{\eta}\right)-1$.

We then present the rolling horizon algorithm as follows.

Step 1. Solve each model with horizon $T2$ for periods $T1$, $T1+1$, $T1+2,\ldots,T2-1$.

Step 2. Fix and implement decisions and data for $T1$ to $T1+\eta-1$.

Step 3. Set $T1 = T1+\eta$ and $T2 = T2+\eta$.

Step 4. If $T2 < T$, go back to Step 1. Else stop.

To gain insight into the effectiveness of the rolling horizon approach as a mechanism to decompose the model and to deal with the end-of-planning horizon effect, we investigate the relationship between rolling and planning horizons and the total profit of the fishery.

For example, to solve a $T = 30$-period planning horizon model, we ran $T2 = 10$-period models by fixing and implementing the decisions and data for fixed horizon $\eta = 5$-periods, since the first five periods are more certain and the last five periods are less certain, and also because of the difficulty in solving longer planning horizon models. Figure 1 shows the rolling horizon $T2$ and fixed horizons η of a $T = 30$-period planning horizon. Results for different planning horizon models are shown in Table 4.

		Profit in planning horizon T ($\$\times 10^6$)							
$T2$	η	10	12	15	18	20	24	25	30
10	5	-	-	\$1.51	-	\$1.70	-	\$1.79	\$1.85
12	6	-	-	-	\$1.61	-	\$1.79	-	\$1.88
	Optimum	\$1.06	\$1.31	\$1.58	\$1.75	\$1.88	\$1.98	\$2.12	\$2.30

Table 4: Profit of different planning horizons for different rolling and fixed horizon.

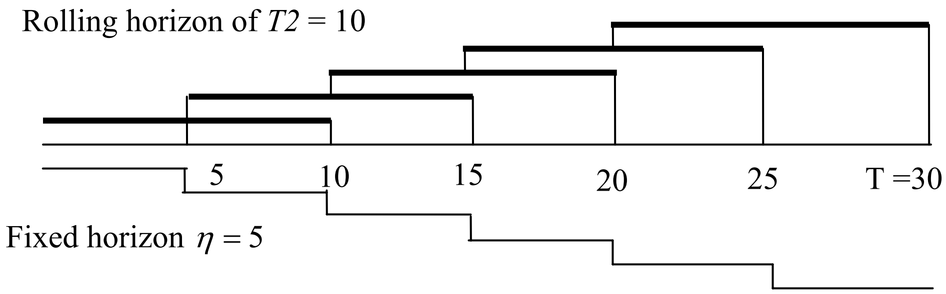

Figure 1: Fixed and rolling horizon for a 30-period planning horizon.

Keeping parameters unchanged, we solved the same 30-period model by rolling a $T2 = 12$-period model by fixing every $\eta = 6$ periods. Results are shown in Table 4. A 12-period model fixed every six period yields a better solution than a rolling horizon of 10-period, because, at the first 12-period solution, the model allocates labour for a longer horizon than the 10-period, and so the idle time at the latter part of the 30-period planning horizon was less than that of the10-period rolling horizon.

Also at the beginning of the rolling horizon, the fishery has a lot of fish quota, so the initial model used more labour for fish processing. Since we fixed this labour for the later horizons which got fewer quotas, the later periods yielded less profit and higher idle time, which creates the end-of-planning horizon effect. To cope with this, the fishery needs to pay more attention to labour allocation. This can be done by setting the average amount of labour used in the entire horizon. For this, the fishery could calculate the total quota available for the entire horizon and calculate the approximate labour hour require for processing per kilogram of fish quota. Suppose the total available quota is Q kilograms and the required labour time for processing a kilogram of raw fish is approximately h hours (averaged over all fish and quality types). Therefore, the required time for processing Q kilograms of raw fish is $Q \times h$ hours.

Hence, for a T periods planning horizon in each period the fishery is required $\frac{Q \times h}{T}$ hours of labour time. However, we can not be sure that the entire quota Q will be used in this horizon.

Instead, the fishery can allocate labour in another way. In this way, the rolling horizon model needs to be solved twice. At the first time, the model will allocate the labour hours for the first rolling model, and then fix the regular labour for the models of the later parts of the rolling horizon. We observe the actual labour used both regular and overtime, and also idle times. Based on the actual labour used, the fishery can set the average labour per period for the entire horizon.

For example, we first ran a T= 30-period planning horizon of the original problem with
$T2$ = 10 and η = 5, which yielded a profit of $1,859,278. The total labour actually used was 32650 hours. We then set the average labour time (32650 / 30) = 1085 for each period of the entire planning horizon; in the second run, it yielded a total profit of $2,087,490 with a 12.3% increase in the profit, but still 9% less than the direct solution of a 30-period problem.

We do the same experiments with IFPMS, IFPML, and IFPMXL. The results are shown in Table 5. From these problems, we notice that these solutions are still 11% to 16% less than the direct solution profit of 30-period models.

Problem	T	$T2$	μ	Direct solution profit ($)	Profit in 1st run ($)	Profit in 2nd run ($)	% change in profit
IFPMS	30	10	5	$1,874,130	$1,436,371	$1,551,080	+7.9
IFPML	30	10	5	$2,550,260	$1,849,005	$2,266,360	+22.6
IFPMXL	30	10	5	$2,546,817	$1,717,530	$2,358,950	+37.3

Table 5: 30-period planning horizon with 10-period rolling horizon for IFPMS, IFPML, and IFPMXL.

In this section, we examined a rolling horizon approach to deal with the large problem size, catch data forecasting, and the end-of planning-horizon effect. We found from the solutions of the four different problem instances that the rolling horizon approach was about 9% to 16% far from the optimum. Therefore, planning for overly short planning horizons can be detrimental to the profitability of the firm.

Rather than allocating labour by period, we could choose to allocate available quota, as we will do in the next section.

4.2 Effect of quotas on the profit

In this section, we analyse the impact of quota allocations on the total profit of the fishery. We also discuss the impact of quotas on the length of planning horizons.

In this experiment, we increase the available quotas of each species and stock area by 10% at a time up to 40% and solve the IFPM for different planning horizon models. The results for up to 20% are shown in Table 6. When the available quota is increased, we expect the number of trips will increase, since the trawler will get more fish to catch and as a result the landed fish will be increased. Consequently the profit of the fishery will be increased.

PH↓		% Change in available quota			
		10	20	-10	-20
5	% change in Profit	0	0	0	0
	% change in Trawler's trip	0	0	0	0
	% change in Landed fish	0	0	0	0
10	% change in Profit	0	0	0	0
	% change in Trawler's trip	0	0	0	0
	% change in Landed fish	0	0	0	0
15	% change in Profit	4.3	7.2	-6.4	-9.1
	% change in Trawler's trip	0	0	-2.4	5
	% change in Landed fish	3.7	6.3	-2.9	-6.4
20	% change in Profit	4.3	7.9	-7.2	-12.9
	% change in Trawler's trip	0	4	-3.2	8
	% change in Landed fish	3.0	5.2	-3.3	-5.7
25	% change in Profit	5.0	9.6	-5.1	-12.4
	% change in Trawler's trip	0	3.3	-2.2	6.6
	% change in Landed fish	1.8	4.3	-4.0	-5.8
30	% change in Profit	4.8	9.6	-4.4	-11.6
	% change in Trawler's trip	0	2.9	-3.8	2.9
	% change in Landed fish	1.9	4.4	-5.0	-4.9

Table 6: Effect of the change quotas

The results show that as the available quota is increased, the number of trawlers' trip taken, increases and as a result the amount of landings and total profit of the fishery increases. The percentage change in profit, trawler's trip, or landed fish is defined as

100 × (Profit or trawler's trip or landed fish base line capacity – Profit or trawler's trip or landed fish with a change in capacity) / (Profit or trawler's trip or landed fish with base line capacity). For example, for 20% increase in quota, the number of trawler's trip increased by 4%. We calculate it as 100 ×(26-25)/25 = 4%.

A 10% increase in the available quota results in an average increase of 1.7% in the amount fish landed. We observe an average increase in the profit is 3.6%. A 20% increase in the available quota results in an average increase of 1.7% in the number of trawlers' trip taken, resulting in a 3.3% increase in the amount fish landed. We observe an average increase in the profit of 5.7%.

We also reduced the available quota by 10% to 40% and observed the impact on the profit. Partial results are shown in Table 6. A 10% reduction in the available quota results in an average decrease of 2.5% in the number of trawlers' trip taken, and resulted in a 1.9% decrease in the amount of fish landed. We observe an average decrease in the profit of 3.9%.

Changing the available quota, we solve the three other different problems IFPMS, IFPML, and IFPMXL. The results were consistent with that of the original problem.

Changing the planning horizon length and keeping other parameters constant, we solved 5, 10, 15, 20, 25, and 30-period models of the original problem, and noticed that a longer planning horizon had a lower average profit per period (Table 7). Since the fish quota is reduced by the amount of raw fish caught, the longer planning horizon models gets lower average quota per period. If the available quota of a fish species finishes during a trawler's trip, then the trawler comes back even if it is not full.

Planning horizon	Beginning and final inventory raw (Kg.)	Fishing cost / period ($)	Regular labour / period (h)	Labour cost / period ($)	Revenue / period ($)	Inventory cost ($)	
						Raw	Product
5	103,456	10,400	1421	28,423	144,652	1,147	129
10	104,481	11,000	1459	29,188	147,732	767	199
15	107,385	12,133	1398	27,964	146,549	721	264
20	85,943	12,175	1189	23,777	130,860	524	216
25	117,140	12,480	1078	21,573	120,707	980	261
30	95,567	12,567	1034	20,694	110,876	564	212

Table 7: Inventory, cost, labour, and average profit per period for different planning horizon models.

The average profit per period of a 5, 10, or 15-period model is similar. That is, a 10-period model approximately yields a profit of two 5-period models, and a 15-period model yields a profit of three 5-period model. But a 20, 25, or 30-period model does not yield profits of 4, 5, or 6 times the 5-period model. Similarly a 20 or 30-period model does not yield a profit of twice or thrice a 10-period model, because for the longer planning horizon the model gets less average quotas and so the model uses fewer regular labour for each period and as a result the fishery process less product resulting in less profit as we will see in the following experiment.

We solve the first 10-period horizon model which yields a total profit of $1,065,775 and uses 1459 hours of regular labour per period. We reduce the quota by the amount of fish caught during this horizon but allow the model to decide the amount of regular labour to be used per period. We also fix the initial quota obtained from the first 10-period model. We observe that the 2nd and 3rd 10-period model uses 901 hours and 586 hours of regular labour per period with no idle hours and yields total profit of $636,262 and $281,318 respectively. The total profit from these three 10-period models is $1,983,355 where as a direct 30-period model yields a total profit of $2,300,871. We also notice that the three 10-period models use 12, 11 and 7 trips respectively which in total are 30 trips. A direct 30-period model use 35-period trips. Results are shown in Table 8. Similarly, we do the same experiment with two 15-period models one after another and observe that the 2nd 15-period horizon model yields lower profit and produces idle time. From these experiments, we found that the quota allocation is important. So we observe the effect of smooth quota allocation on the profit in the following experiment.

Planning horizon	Profit	Number of trawler trips	Amount of fish landed (kg)
1st 10-period	$1,065,775	12	573,705
2nd 10-period	$636,263	11	451,440
3rd 10-period	$281,318	7	320,512
Total of three 10-periods	$1,983,356	30	1,354,657
30-period (direct)	$2,300,871	35	1,530,540

8. Table 8: Comparison of three 10-period horizons to a 30-period horizon.

In this experiment, we allocated one-third of the total available quota for 30-period horizon to each of the three 10-period horizons. Each 10-period model yielded a profit of $724,007. So, three of these planning horizons yielded an average profit of $72,4007 per period which is close to the average profit of a direct 30-period model ($76,696 per period). The total profit from these three 10-period models is $2,172,021 which is the closest profit to a 30-period direct solution profit ($2,300,871) resulting in only 5.6% lower profit than the direct solution of a 30-period model.

So we conclude that, the fishery could reduce the solution gap by smoothing allocation of available fish quota with a rolling horizon approach but the result is still about 5% far from the optimum. We also observed that, smoothing the quota allocation results in higher profit than that of smoothing labour.

4.3 Effect of trawlers' capacity on the profit

In this section, we analyse the impact of trawlers' capacity on the total profit of the fishery. When the capacity of the trawlers is reduced, we expect the number of trawlers' trip will be increased, and also the amount of landed fish will decrease because the trawler will be able to store less fish with decreased capacity. As a result the total profit of the fishery should be decreased. The results are shown in Table 9.

PH↓		% Change in trawlers' capacity						
		-10	-20	-30	-40	-50	10	40
5	% change in Profit	-4.7	-11.8	-22.4	-33.2	-46	4.3	14.8
	% change in Trawler's trip	0	0	0	0	0	0	0
	% change in Landed fish	-4.9	-12.3	-23.3	-34.2	-45.2	4.9	19.7
10	% change in Profit	-5.9	-13.2	-23.6	-34.3	-46.8	4.9	15.9
	% change in Trawler's trip	0	0	0	0	0	0	-8.3
	% change in Landed fish	-6.2	-14.4	-25.1	-35.8	-46.5	6.3	21.2
15	% change in Profit	-3.5	-7.6	-12.4	-24.7	-37.5	2.8	8.2
	% change in Trawler's trip	0	0	5	5	5	-5	-15
	% change in Landed fish	-2.6	-8.5	-15.5	-27.6	-39.6	4.1	13.3
20	% change in Profit	-3.9	-6.5	-14	-20.2	-32.2	3.7	6.4
	% change in Trawler's trip	0	4	4	8	8	-4	-12
	% change in Landed fish	-3.9	-7.3	-17.3	-24.5	-37.1	4.5	15.9
25	% change in Profit	-4.4	-8.8	-14.5	-21.6	-30.2	5.3	5.1
	% change in Trawler's trip	6.7	6.7	10	13.3	16.6	-3.3	-16.7
	% change in Landed fish	-5.7	-9.8	-17.8	-25.6	-35.3	7.1	14.5
30	% change in Profit	-3.6	-7.1	-12.6	-21.7	-30.4	4.7	3.7
	% change in Trawler's trip	5.7	8.6	8.6	11.4	14.3	-5.7	-20
	% change in Landed fish	-4.7	-9.1	-14.8	-25.6	-35.3	6.1	12.9

Table 9: Effect of the variation of trawlers' capacity

The percentage change in profit, trawler's trip, or landed fish is defined as

100 × (Profit or trawler's trip or landed fish with base line capacity – Profit or trawler's trip or landed fish with capacity reduction) / (Profit or trawler's trip or landed fish with base line capacity).

The results show that as capacity is reduced the number of trips increases and the amount of landed fish decreases as we expected. As a result the total profit of the fishery decreases. A 10% reduction in the trawler capacity results in an average increase of 2.1% in the number of trips taken. The amount of landed fish on the other hand decreases 4.7%. As a result, we notice an average decrease in the total value of the objective function of 4.4%.

A 50% reduction in the trawler capacity results in an average increase of 7.3% in the number of trips taken. The average amount of landed fish on the other hand decreases 41.5%. As a result, we observe an average decrease in the total value of the objective function of 37.2%.

We also increased the trawlers' capacity by 10% at a time up to 40% and observed the impact on the profit. Results are shown in Table 9.

Changing the trawlers' capacity, we solve the three other different problems IFPMS, IFPML, and IFPMXL. The results were consistent with that of the original problem.

6. Conclusion

In this paper, we apply a rolling horizon approach for the solution of a 30-period planning horizon model. We also presented the structure of the IFPM and presented the computational time to show it was hard to solve. From the four different problem instances, we found that the classic approach of a rolling horizon was ineffective in the sense that it took long time to solve the 30-period IFPM, and reduced profits significantly. The rolling horizon was intended to reduce the size of the problem to make it solvable. But it was proved to be ineffective to reduce the problem size and either took longer time to solve the longer planning horizon problems or the solutions were far from the optimum. So this is not a good way to decompose the IFPM and if the management tries to operate that way, they will be about 9% far from the optimum. We also found that the smoothened allocation of quota can reduce the solution gap but still about 5% from the optimum.

This still leaves the problem of solving a large MILP for a long planning horizon. We are currently working on column generation approaches to help solve these large models quickly.

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